

Philosophical  
PRINCIPLES  
OF  
RELIGION.

PART II.

CONTAINING  
The NATURE and KINDS  
of *Infinities*; their *Arithmetick*  
and *Uses*, and the *Philosophick*  
*Principles* of *Reveal'd* RELI-  
GION, now *first* published.

By GEORGE CHEYNE, M. D. and F. R. S.

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PART II.



The N. and Kinds  
of human beings  
and the different  
forms of Art, &c.  
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By George GORDON, M. A. &c.

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# THE PREFACE.

To the Second PART.

**B**Efore the *Reader* takes the Trouble to enter on the *Second Part*, it's fit he be apprised of a few Things, that may prevent his mistaking my Meaning, in the more difficult Propositions, or may enable him to run thro' the whole with more Pleasure.

I. The *first Chapter* is intended only, as an Introduction to the two subsequent, to the *third* especially; and for clearing up and demonstrating in the most familiar and easy Way, the Nature and Properties of *relative Infinites*. The Foundations of the *Arithmetic of Infinites*, and the Structure built thereon, published by me in the *former Edition* of this Work, having been doubted of, or mistaken by some. The whole now, I hope,

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## The P R E F A C E.

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is clear and unquestionable. I have borrowed a few Things from the *third Chapter*, for the Ease of those, who either might not have Humour and Leisure, or might not have apply'd themselves sufficiently to such Studies, as to be able to go quite through *that Chapter*. And yet might be content to see this curious Speculation established here.

II. The *second Chapter* contains many Particulars of Weight and Moment. The *Foundation* is this. The *supreme Creator* of all Things, and the whole *System* of Creatures, from the highest *Seraphim* down to *brute Matter*, are here together considered as it were an *infinite Cone*, (like the *Shadow* of the dark Side of the *Earth*, circumscribed by the Light of the *Sun* in the empty Spaces of our *System*,) whose *Base* is the *supreme and absolute Infinite*, the Origin of the Being and Faculties of all created Things; and its *Body*, is the whole *System* of Creatures, from the highest *spiritual Intelligence*; descending in a perpetual Subordination, and continual Scale, down to *brute Matter*; or if there be any Creature lower than this. It is true, in this *Metaphor* or *Resemblance*, the *Base* is to be supposed at an *absolutely infinite* Distance, from the Body of the *Cone*, (as the *Sun*, whose Rays define the dark *Cone* of the *Earth's* Shadow, is distant from the *Earth*.) But then, as all the *Sections*  
in

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in a *Cone*, parallel to the *Base*, are similar to the *Base*, and to each other. So in this *perpetual Scale* of Creatures, considered in one View, together with their Creator, every *Species* and Set of Creatures is similar to the *Base*, and to every other *Species* and Set of Creatures, from the highest to the lowest, *i. e.* every *Species* and Set of Creatures of a higher Order, has the great *Lineaments*, and *prominent Out-lines* of their *Base*, the Origin of all Being and Perfections, more strongly, clearly, and largely represented and express'd on, and by them, and every *Species* and Set of Creatures of a lower Order, has the same *Lineaments* and Characters represented and express'd on, and by them, but in a more weak, more faint, and more contracted Manner. And since *Life*, *Activity*, and *Fecundity*, are among the most *universal*, *primitive*, and *original* Qualities of the *Base*, the *Source*, and *Origin* of all Being and Perfections: So every *Species* of Creatures and each Individual of every *Species*, must in a higher or lower Degree, according to their Rank in the Scale of Existence, partake of those *primitive* and *original* Qualities. If this *Principle*, and *Foundation*, thus shadow'd out, may be allow'd me, and sure methinks it is evident from the Nature of Things *à Priori*, from all Experiments and Observations hitherto

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made on our material *System* of Things, *a Posteriori*, and even from the most genuine and simple Reflections of our Minds within our selves. Then it will follow;

I. That there is a perpetual *Analogy*, (physical not mathematical) running on in a Chain, thro' the whole *System* of *Creatures*, up to their *Creator*.

II. That the Visible are Images of the Invisible, the Sensible of the Insensible, the *Ectypical* of the *Archiotypical*, the *Creatures* of the *Creator*, at an absolutely infinite Distance.

III. That the Arguments from the *Attributes* of the *Creator*, to the Qualities of the *Creature*, with due regard to the absolutely infinite Distance, is just and conclusive, & *vice versa*.

IV. That, as there are *Objects* intirely opposite and *disparata*, so there must be Faculties in intelligent *Creatures*, suited to those different *Objects*, differing according to the Diversity of the *Objects*.

V. That if *Gravitation* be the Principle of *Activity* in *Bodies*; that of *Re-union* with their Origin, must by this *analogical* Necessity be the Principle of *Action* in *Spirits*.

VI. That *material Substances* are the same with *spiritual Substances*, of the higher Orders, at an infinite Distance, or that *materi-*  
*al*

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*al Substances* are *Spiritual Substances* infinitely condensed or contracted, since in the *Scale of Existence*, the *first* are supposed at an *infinite* Distance from the *latter*.

VII. That there is some *Analogy* between the *Constitution*, *Temperaments* and *Complexions* of *Spiritual Beings*; and the known different *Textures*, *Elements*, and *Faculties* of *material Substances*.

THESE I think, as they are necessary Consequences from the preceding Principle, so they are the main Pillars, and some of the principal Propositions of this *second Chapter*, which, if understood and granted, every Thing else will either be easily receiv'd, or may be safely rejected, without any Hazard to the main *System*.

AFTER all, seeing my whole Intention and Design, in advancing and publishing these *Speculations*, was to beget in the Minds of Men, noble, generous, and magnificent Sentiments, of *God* and *his Works*, that, thereby they might be more powerfully engaged, to love, adore, and serve *Him*. To convince them of the *Degeneracy* and *Corruption* of the whole Race of Mankind; of the Necessity of expanding and cultivating their *superior* Faculties, by a faithful Obedience to the divine *Attraction* and *Drawings* in their Hearts; and thereby, of begetting in their Souls, *Chari-*



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ty, or the *pure Love of God*, and of all his Images in a proper Subordination: All which can by no other Means be brought about, but by a careful copying after, and imitating the *Model and Pattern* the BLESSED JESUS has set us in his Life and Doctrine. I say, since this was my whole End and Aim, in advancing and publishing these *Speculations*. If any Person shall think fit to contravert them, he may do it very safely for me. For being satisfied, in the Honesty and Simplicity of my Intentions, and of the Use and Benefit these Speculations have been to my self for these Ends and Purposes. I am firmly resolved, not to spend my Time in idle Disputes. If others differ with me, about the Truth and Reality of these *Speculations*, or their usefulness to, and influence on the Ends proposed, or are not dispos'd to relish or receive them, they may let them alone or reject them, it is equal to me. All I shall be ever prevail'd on to do, in such a Case (excepting always, in Case those who either are my *lawful Superiors*, or whom I look on my self obliged in Conscience to obey, shall command otherwise,) shall be ; to amend, alter, or retract what I shall be perswaded is amiss in the future *Editions* of this Work, if it shall have any more.

III. The



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## The P R E F A C E.

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III. The *third Chapter* is what the *reverend* and *ingenious* Mr. *John Craig* sent me about seven Years ago, when I desired him (being low in my Health, and otherwise engaged) to write me down his Thoughts on, correct or alter, what I had formerly published on this Head in the *first Edition* of this Work, in order to a second Edition. I have altered or added nothing, but one *Note* before his *Additions*, and that in *Italick* Characters.

To conclude, if any Person, by either of the Parts of this Work, shall be moved to adore, worship, or love the *lovely and adorable Author of his Being*, (who is wonderful in all his Works, and great in the least,) I say, if any one shall be wrought on thereby to love him more, or serve him better, I shall have the whole Reward of my Labour, having intended it solely for his *Glo-ry* and the *Good* of my fellow Creatures; and having I hope, in the whole, and each single Part, as far as my Weakness and Corruption wou'd permit, disengaged myself from all *sinister* Ends, from all *Fraud, Malice, Vain-glory*, and *Hypocrisy*.

T H E

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**C O N T E N T S**  
 O F T H E  
**S e c o n d P A R T.**

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**C H A P. I.**  
**Of the Nature and Kinds of Infinites,**  
**of some of their respective Quali-**  
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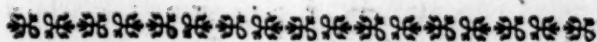
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## PART II.



## PART II.



### CHAP. I.

*Of the Nature and Kinds of Infinites. Of some of their respective Qualities, and of a New Arithmetick of Infinites.*



H A T we may reason, as cautiously as possibly we can, about Matters so intricate, and so far remov'd from the common Way of Thinking, as the Nature and Qualities of *Infinites*, and the other Subjects of this Chapter are; we shall begin with *Definitions* and *Axioms*, and proceed to some general *Propositions*, demonstrated after the plainest manner,  
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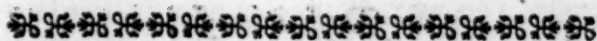
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Prob. III. *Having the second Terms, in two*  
*geometrical Series, whose first Term is in*  
*both Unity, together with the Number of*  
*Terms in the other, to find the other Se-*  
*ries that shall make the last Term in both*  
*equal.* 180  
Prop. IV. *A general Method to find in any*  
*Hyperbola the Points where the Curve*  
*meets with its Asymptots.* p. 182; and 189  
*An Answer to Mr. Varignons Reflections*  
*upon Spaces greater than Infinite.* p. 185

## PART II.





## PART II.



### CHAP. I.

*Of the Nature and Kinds of Infinites. Of some of their respective Qualities, and of a New Arithmetick of Infinites.*



HAT we may reason, as cautiously as possibly we can, about Matters so intricate, and so far remov'd from the common Way of Thinking, as the Nature and Qualities of *Infinites*, and the other Subjects of this Chapter are; we shall begin with *Definitions* and *Axioms*, and proceed to some general *Propositions*, demonstrated after the plainest manner,  
B till

till we obtain *Principles* to found our consequent Reasonings on ; and then draw such Corollaries, from the several Parts, or from the whole, as arising necessarily from them, may be of use to ascertain some Speculations advanced in the foregoing *Treatise* ; or may otherwise help to conduct the Understanding in those other *Sciences*, where they may find a Place.

*Definition I.*

QUANTITY is what may be increas'd or diminish'd.

THO' this *Definition* may not exhaust the *metaphysical* Nature of *Quantity* ; yet it points out that *Quality* in it, that is here chiefly regarded ; for ev'ry *Quantity* may be increas'd or diminish'd, and that continually, as shall be afterwards shown

*Definition II.*

A BODY is an extended, impenetrable, passive, divisible, unintelligent Substance.

This *Definition* also, tho' it exhaust not the internal Essence and intimate Nature of Matter, yet it sums up its sensible and most constant Qualities, by which it is distinguished from every thing else.

*Defi-*

## Definition III.

THE inherent *Principle* of *Activity*, in the great Bodies of the Universe, is *Gravitation* or something *analogous* thereto.

THO' I am perfectly convinced, from the Simplicity and Uniformity of the divine Nature, and of all his Works, that there is some one great and universal *Principle*, running through the whole *System* of Creatures *analogically*, and congruous to their relative Natures; which is the same in all Bodies great and small, and the *Origin* of all their natural Actions upon one another, with regard to their different Circumstances; and that there is not a different *Principle* for the natural Actions of the *lesser Bodies* from that which is the *Principle* of the natural Actions of the *greater Bodies* of the *Universe*, but one and the same *Principle* in both, acting differently in different Circumstances. Yet since *Gravitation*, or something *analogous* thereto, seems necessary for accounting for the constant and regular Motions, and Actions upon one another, of the *great Bodies* of the Universe; *Gravitation* or something *analogous* thereto must be a necessary Consequence in the *greater Bodies* of the Universe, of this more universal Principle, and the *Origin* of the Activity of *Bodies*.

*Definition IV.*

A SPIRIT is an extended, penetrable, active, indivisible, intelligent Substance.

*Body* and *Spirit* are in ev'ry other Quality opposite, except in *Extension*; therefore as the foregoing *Definition* of *Body* summs up its sensible and most constant Qualities, so to assign the *Definition* of *Spirit*, there was nothing to be done, but to joyn the opposite Qualities of *Body* to that of *Extension* or extended Substance.

*Definition V.*

THE *Principle* of Action in spiritual Subsistences is, or ought to be, that essential one of REUNION with the *Origin* of their Being, impress'd on ev'ry Individual of this Rank of Creatures.

THE universal *Principle* of *Action*, mentioned in the third *Definition*, that runs through all the *System* of Creatures, must *analogically* be carried through ev'ry Individual of *spiritual* Beings, and can be nothing but this essential *Principle* of REUNION with the *Origin* of their Being, as shall be afterwards demonstrated at large.

*Defi-*

*Definition VI.*

A *finite Quantity* is that, of which the Bounds or Limits, beyond which it cannot reach, are *assignable*.

THUS a Line is finite, when both its Extremities are given, or the Points which are its Limits, beyond which it cannot reach, are *assignable* : An *Area* is finite, when its terminating Lines are *assignable* ; a *Solid* is finite, when its terminating Planes are *assignable* ; a *Number* is finite, when the Unities (which are its Limits) of which it consists, or the Bounds beyond which it cannot reach, are *assignable*.

*Definition VII.*

AN *infinite Quantity* (in its simplest Nature and lowest Degree) is that, some one or more of the Limits or Bounds of which, beyond which it cannot reach, are not *assignable*.

THUS a *right Line*, one or both of whose Extremities are not *assignable*, or the Points beyond which it cannot reach, are not *assignable*, is an *infinite right Line*. An *Area*, one or more of whose terminating Lines are not *assignable*, is an *infinite Area* ; a *Solid*, one or more of whose terminating Planes



## 6 Philosophical Principles

are not *assignable*, is an *infinite Solid*; a Number, increasing continually, whose last Increase is not assignable, or the Bounds, beyond which it cannot reach, are not *assignable*, will at last make an infinite Number.

### *Scholium.*

AN infinite *Number* may be suppos'd to be generated by the perpetual Addition of a finite Number to it self. Thus  $1 + 1 + 1 + 1 + 1$ , &c.  $a + a + a + a + a$  &c. become infinite. Or, it may be supposed to be generated, by the perpetual Addition of finite Numbers, increasing in a regular Progression, and in one constant Proportion one to another. Thus  $1 + 2 + 4 + 8 + 16$ , &c. (where the finite Terms perpetually increase in the *ratio* of 1 to 2.) and  $1 + a + a^2 + a^3 + a^4$  &c. (where the finite Terms perpetually increase in the *ratio* of 1 to  $a$ ) become infinite. And it is the same in all other infinite *Series*, regularly generated; or lastly, the infinite Number may be supposed to be generated by the perpetual Addition of finite Numbers, in no certain Proportion one to another, nor in any regular Progression, such as  $7 + 1 + 30 + 5 + 2 + 25$  &c. Of these last Kinds of Infinites we have here no Consideration; for being of no constant or regular Nature, but merely casual and fortuitous, they

they can afford no *Medium* for reasoning. The principal Design of this Chapter, so far as it concerns these two Kinds of Infinites, is to find out a *Method* for resolving the second Kind of Infinites into the first, when it is possible ; in order then to obtain a just Notion of these Infinites, let us first distinguish Infinite in general, into *relative* or *creaturely* Infinite, and *Supreme* or ABSOLUTE Infinite (of which the first is but a created Image or Picture, as will be afterwards shewn) let  $\odot$  stand for Finite in general, and  $\infty$  stand for Infinite in general as they respect Numbers ; then  $\odot 1$  and  $\infty 1$  will be finite and infinite (as they respect Numbers) of the lowest Degree and simplest Nature ; Unity being the simplest Number.

*Definition VIII.*

*Relative Infinite* (in its simplest Nature and lowest Degree) is an infinite Quantity, as it stands related to a given Finite, by the perpetual Addition of which to it self it is generated.

THUS  $\infty 1$ . is a *relative Infinite*, as it stands related to 1, a given Finite, by the perpetual Addition of which to it self, it is generated ; that is,  $\infty 1 = 1 + 1 + 1 + 1 + 1 \text{ \&c.}$  And  $\infty a$  is a *relative Infinite*,

## 8 Philosophical Principles

as it stands related to the given Finite  $a$ , by the perpetual Addition of which to it self it is generated, or  $\infty a = a + a + a + a + a$  &c.

*Scholium.*

IN the same Relation, that relative Infinite stands above a given Finite in *ascending*, in the same may another Quantity be supposed to stand below it in *descending*, in which Case, we shall have a *relative infinitely great* Quantity in *ascending* above the given Finite; and a *relative infinitely little* Quantity in *descending* below it. So that relative Infinite in general may be aptly distinguished, in respect of the given Finite, into *relative infinitely great*, and *relative infinitely little*. For brevity sake, we shall call the first *relative Infinite*, the second *relative Nothing*.

### Definition IX.

*Relative Nothing* (in its simplest Nature and lowest Degree) is an *infinitely little* Quantity, as it stands related to a given Finite, by the perpetual Subtraction of which from it self it is generated. Let  $o$  stand for relative Nothing.

THUS  $o$  is a *relative infinitely little* Quantity, as it stands related to Unity, by the per-

perpetual Subtraction of which from it self, it is generated; that is  $01 = 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 \text{ \&c.}$  and  $0a$  is an infinitely little Quantity, as it stands related to the given Finite  $a$ , by the perpetual Subtraction of which from it self, it is generated; that is  $0a = a - a + a - a + a - a \text{ \&c.}$

### Definition X.

AN *indefinite Quantity* (in its simplest Nature and lowest degree) is some mean Proportional, between Finite, and relative Infinite or relative Nothing.

FOR, as in *descending* from 1 to 0, we do not immediately slip from finite to relative Nothing, but must necessarily pass through the intermediate Steps  $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \text{ \&c.}$  in Arithmetical Progressions; and in *ascending* from 1 to  $\infty$  we must pass through the Steps between both 2, 3, 4, 5,  $\text{\&c.}$  in the same Kind of Progressions. So in the *Geometrical* Progressions, in *descending* from 1 to 0, we must pass through these mean Proportionals  $^2\sqrt{0}$   $^3\sqrt{0}$   $^4\sqrt{0} \text{ \&c.}$  and in the same Progressions, in *ascending* from 1 to  $\infty$ , we must pass through the mean Proportionals  $^2\sqrt{\infty}$   $^3\sqrt{\infty}$   $^4\sqrt{\infty}, \text{ \&c.}$  and these we call *Indefinites*. Thus in *Geometry*, if we cou'd imagine a *Circle*, drawn upon the summ of a finite right Line representing 1, and an infinite right Line representing





$$+ \frac{1 \times m + 1 \times 2m + 1}{m \times 2m \times 3m} + \frac{1 \times m + 1 \times 2m + 1 \times 3m}{m \times 2m \times 3m \times 4m}$$

$$+ \frac{3m + 1}{4m} + \mathcal{C}. \text{ Now as } \infty 1 = 1 + 1 + 1 +$$

$$1 + 1 \mathcal{C}. \text{ and } 0 1 = 1 - 1 + 1 - 1 + 1 - 1, \mathcal{C}.$$

and  $\infty \sqrt{\infty} 1$ . are of the simplest Nature, so also are they of the lowest Degree. The superior Degrees being generated of  $\infty 1$ , after the same Manner that  $\infty 1$  is of 1, for if we add  $\infty 1$ , perpetually to it self, we shall have a *relative infinitely great* Quantity, of the simplest Nature in its Kind, but of a higher Degree, viz.  $\infty 1 + \infty 1 + \infty 1 + \infty 1 \mathcal{C}. = 1 + 1 + 1 + 1 + 1 + 1 + 1 \mathcal{C}. + 1 + 1 + 1 + 1 + 1 \mathcal{C}. + 1 + 1 + 1 + 1 + 1 \mathcal{C}. \mathcal{C}. =$  (Since a perpetual Addition of any Quantity to it self, is equal to a Multiplicati-

$$\text{on by } \infty 1) \infty 1 \times 1 + 1 + 1 + 1 + 1 + 1 \mathcal{C}. = \infty 1 \times \infty 1 = \infty^2. \text{ So } \infty a + \infty a + \infty a + \infty a \mathcal{C}.$$

$$= \infty \times a + a + a + a + a + a \mathcal{C}. = \infty \times \infty a = \infty^2 a. \text{ After the same manner } 0 1 = 1 - 1 + 1 - 1 + 1 - 1 \mathcal{C}. \text{ If subtracted perpetually from it self, it becomes } 0 1 - 0 1 + 0 1 - 0 1 \mathcal{C}.$$

$$= 1 - 1 + 1 - 1 + 1 - 1 \mathcal{C}. - 1 - 1 + 1 - 1 + 1 - 1 \mathcal{C}. + 1 - 1 + 1 - 1 + 1 - 1 \mathcal{C}. - 1 - 1 + 1 - 1 + 1 - 1 \mathcal{C}. + \mathcal{C}. =$$
 (since a perpetual Substraction of a Quantity from it self is the same with a Division by  $\infty$ )  $\frac{1 - 1}{+ 1}$

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$$\frac{+1-1+1-1 \text{ \&c.}}{\infty} = \frac{0}{\infty} = \frac{1}{\infty} \times 0 = 0 \times 1 = 1$$

$+1-1+1-1 \text{ \&c.} = 0^2$ . Thus we have a relative Nothing less than 0, but of a superior Degree; for as *relative infinitely great* Numbers increase in their Value, by being raised to superior Degrees, so *relative infinitely little* decrease, because the first perpetually ascend from Finite, the latter *descend* perpetually further from it. And thus all the Degrees, that finite Quantities admit of, may be form'd from relative Infinites and Nothings. And as we have Infinites  $\sqrt[m]{\infty}$  between 1 and Infinite in *ascending*, and  $\sqrt[m]{0}$  between 1 and 0, in *descending*, so *analogous* to their Natures we have the superior Degrees  $\sqrt[m]{\infty^n}$  and  $\sqrt[m]{0^n}$ . Nature in all these Cases admitting of no Bounds nor Limits.

### Definition XI.

*Absolute* or SUPREME Infinite, in a proper Sense, is one, Individual, admitting of neither Increase, nor Diminution, or of any Operation that *mathematical* Quantity is subjected to.

THIS will be better understood afterwards.

*Defi-*

## Definition XII.

*Absolute Nothing* in a proper Sense is neither capable of increasing nor diminishing, nor of any wise altering any *Mathematical* Quantity to which it is apply'd, but stands in full opposition to *absolute* Infinite.

## Axioms.

I. That which is greater or less than any possible finite Quantity, how great or little soever, must be a *relative Infinite*, *Indefinite*, or *Nothing*, and which of all these Three, the given Quantity is, the State of the Case will always determine.

BESIDES, *Infinite*, *Indefinite*, and *Nothing*, relatively considered, we have no *Idea's* of Quantity, and the Definitions of these already given, apply'd to the State of the Case under Consideration, will always determine which of these the Quantity assigned must be.

II. Number being the simplest Measure of Quantity, and a proper Unity being the Measure of all Number, a proper Unity is the Measure of all Quantity.

THAT Unity is the Measure of all Integers is evident: And in Fractions, the *Denominator* determines the proper Unity, whereof



yet you shall never reach the Extremity *B*,  
 since the  $\frac{m}{n}$  Part is still less than the whole  
 Remainder. That is, you may take out  
 $\frac{m}{n}$  Parts in a certain Proportion out of the  
 given Quantity *AB* perpetually, that is, the  
 Quantity *AB* may be divided in *Infinitem*.  
 q. e. d.

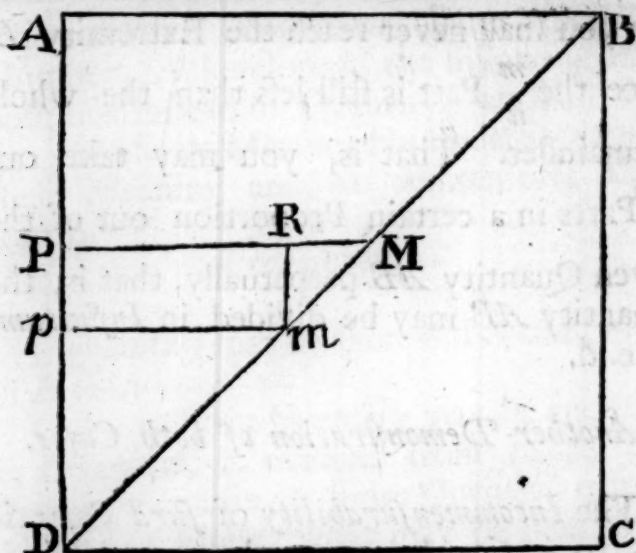
*Another Demonstration of both Cases.*

The *Incommensurability* of *surd* Quanti-  
 ties to *rational* ones, as they are call'd, is  
 a full Demonstration, that *Quantity* may be  
 increas'd or diminish'd in *Infinitem*.



For





Let  $ABCD$  be a Square, whose *Diagonal* is  $DB$ , putting the Side  $AD=1$ . Then is  $AD$  to  $DB$ , as 1 to  $\sqrt{1}$ . Take in the Side  $AD$ , apart  $Pp$  the least possible draw  $PM$ .  $pm$ . parallel to  $AB$  and  $Rm$  parallel to  $AD$ . Since the Triangles  $BAD$  and  $MRm$  are similar,  $Rm$  will still be to  $mM$  as 1 to  $\sqrt{1}$ . therefore it is impossible to find in  $AD$  a Part how small soever, that taken, any finite Number of Times, how great soever, shou'd be equal to  $mM$ . So that let  $AD$  be divided into Parts how small soever, and how many soever, yet still  $DB$  may be further divided. That is *per Def. 5.* and 4. and *Axiom 1.*  $DB$  may be divided in *Infinitem*, and  $AD$  increas'd in *Infinitem*. q. e. d.

Corol.

## Corollary I.

HENCE it is evident, that to assign the *absolutely* greatest *relative Infinite*, or the *absolutely* least *relative Nothing* is a plain Contradiction, seeing both these are still *mathematical* Quantities (as is plain from *Def.* 1. 4. & 5, and shall be afterwards further demonstrated) and so by this Proposition, are capable of further Increase or Diminution; and so the assign'd can neither be the *greatest* nor *least, absolutely*.

## Corollary II.

HENCE, and from *Def.* 4. & 5, we may discover wherein the *specifick* Difference between *Finites* and *relative Infinites* or *Nothings* consists: To wit, in the limited Increase or Diminution of the former, and in the perpetuity of the Increase or Diminution of the latter; for as soon as the Increase or Diminution in these latter stops, they become limited and assignable, and consequently *Finite*; and thereby, no Part of the desired *Infinite*.

C

Coro<sup>l</sup>

## Corollary III.

HENCE it appears, that an Infinite of either Sort is (as to all *arithmetical* Operations on it, with due regard to the Perpetuity of its Increase, or Diminution) of the Nature of an unknown Quantity in *Algebra*. For as in this, from the State of the *Problem*, we perform *arithmetical* Operations on it, as it were known, and thereby we sometimes do, and sometimes do not determine its Value, but by Approximation: So on this, we may perform the like Operations as upon an unknown Quantity, with due regard to its particular Nature, and the State of the *Problem*, and thereby often discover the *specifick Genius* of its Progression, which is always *regular* and *harmonious*, as will be afterwards seen.

## Proposition II.

Unity divided by an *infinite Number* of *Unities* makes the Quotient *relative Nothing*

or  $\frac{1}{\infty} = 0$ .

*Demonstrat.*  $\infty 1 = 1 + 1 + 1 + 1 + 1$  &c. per Def. 4.  $\infty 1 = 1 - 1 + 1 - 1$  &c. per Def. 5. divide 1, by  $1 + 1 + 1 + 1 + 1$  &c. and by the common Operations of *Algebra*, you shall have  $1 + 1 + 1 + 1 + 1$  &c.)  $1 (1 - 1 + 1 - 1 + 1$  &c.  $= 0$ . q. e. d.

Corol-

Corollary I.

FROM hence it is evident, that *Unity* divided by *relative Nothing* is equal to  $\infty$  1. for  $1 \div 1 + 1 + 1 + 1 + 1 \text{ \&c.} = 0$  1  $(1 + 1 + 1 + 1 + 1 \text{ \&c.} = \infty$  1 therefore  $\frac{1}{\infty} = 0$ .

Corollary II.

HENCE also it follows, that  $\infty \times 0 = 1$ . But this may be demonstrated otherwise thus.  $\infty 1 = 1 + 1 + 1 + 1 + 1 \text{ \&c. per Def. 4.}$  Multiply both by 0 and then it is,  $\infty 0 = 0 + 0 + 0 + 0 + 0 \text{ \&c.} = 0 \times 1 + 1 + 1 + 1 + 1 \text{ \&c.}$  but by *Prop. 2.*  $0 = \frac{1}{\infty}$  and  $1 + 1 + 1 + 1 \text{ \&c.} = \infty$  therefore  $\infty \times 0 = \frac{1}{\infty} \times \infty = 1$  q. e. d.

Proposition III.

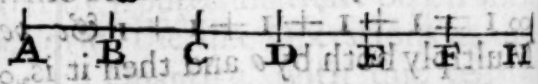
As *Finite* in general is to *relative Infinite* in general, so is *relative Nothing* to *Unity*. That is  $0 : \infty :: 0 : 1$ .

*Demonstrat.* By 1. *Corollary Prop. 2.*  $\frac{1}{\infty} = 0$ . Multiply both by 0 then  $\frac{0}{\infty} = 0 \times 0$  that is  $0 1 = \infty 0 0$  therefore  $0 : \infty :: 0 : 1$ . q. e. d.

## Proposition IV,

*Relative Nothing* is a real mathematical Quantity, and implies the least Part of the Finite, to which it is related or compared.

*Demonstrat.* This is evident from the Generation of *relative Nothing*, assign'd in *Def.* 5. But to demonstrate the Proposition, without regard to this Definition, let *AH* infinitely produced from *A*, be divided into equal Parts *AB, BC, CD, DE, &c.* So that an equal

Part of this Line 

may denote any Number. Supposing  $AB = 1$ , let  $x$  denote any Number, for Example  $x = AB$ ,  $y = Ab$ , then by the common Rules

of Division  $\frac{1}{y-x} = \frac{1}{y} + \frac{x}{y^2} + \frac{x^2}{y^3} + \frac{x^3}{y^4} \&c.$

Now suppose  $b$  infinitely near to  $B$ , then

$y-x = Bb = x$  &  $\frac{1}{x} = \frac{1}{y-x}$  but  $x = \dot{x} + \dot{x} +$

$\dot{x} + \dot{x} \&c.$  by *Def.* 4. that is  $x = \dot{x} \times \frac{1}{1 + 1}$

$\frac{1}{x} = \frac{1}{\dot{x} \times \frac{1}{1 + 1}} = \frac{1}{\dot{x}} + \frac{1}{\dot{x}} + \frac{1}{\dot{x}} + \frac{1}{\dot{x}} + \frac{1}{\dot{x}} \&c.$  therefore  $\frac{1}{x} = \frac{1}{\dot{x}} + \frac{1}{\dot{x}} + \frac{1}{\dot{x}} + \frac{1}{\dot{x}} + \frac{1}{\dot{x}} \&c.$

But by Supposition  $AB = x = 1$  therefore

$\frac{1}{x} = 1 + 1 + 1 + 1 + 1 \&c. = \infty 1$ . But by

*Corollary* 1. *Prop* 2.  $\frac{1}{\dot{x}} = \infty$  therefore  $\frac{1}{x} = \frac{1}{\dot{x}}$  and



$\frac{1}{x}$  and consequently  $x = 0$ . But  $x$  being a real *mathematical* Quantity, by Def. 1. 0. also must be a real *mathematical* Quantity, and the least Part of Unity to which it stands related or compared. q. e. d.

Corollary I.

SINCE  $\infty$  ascends from the given Finite in the same Manner that  $0$  descends below it, and since  $0$  is a real *mathematical* Quantity, so also must the  $\infty$  be. And as  $0$  is the least relatively below it in its own Order, so is  $\infty$  the greatest relatively above it in its Order, but both below and above these, we may *descend* or *ascend* in a higher Order or Degree, without Bounds or Limits.

Corollary II.

WHEN a *Curve* is said to meet with its *Asymptot*, and when in the common *Hyperbola* we obtain the Area  $\frac{1}{1 - 1}$  or in the

Example proposed we put  $y = x$ , in these and such like Cases we mean only, that in the first Case the *Ordinate* is infinitely little, in the second, we mean the least Part of the *Abciss*, and in the third that  $y$  and  $x$  must differ only, by an *infinitely little* Part of  $x$ , or by  $x$  only; and not that

they are *absolutely* equal, else there wou'd be no Division, and consequently no Quotient, as shall be afterwards shewn.

Corollary III.

SINCE by Corollary I, 2d Prop.  $\infty = \frac{1}{1} =$

$$\frac{1}{1-1+1-1+1} \text{ &c. } = 1+1+1+1+1$$

$$\text{&c. Since also } \frac{1}{1-1} = 1+1+1+1+1$$

&c. as is evident from the common Rules of Division, seeing the two first Terms only effect the Quotient, (all the rest being but Repetitions of the same Terms) and lastly, seeing the same Quotient is obtain'd whether the Divisor be  $1-1$  or  $1-1+1-1+1-1$  &c. the reiterated Multiplication of the Quotient upon the Divisor, producing the same Effect (in the actual Operation of the Division) which soever of the Divisors we choose, we may safely put in common Cases  $1-1$  or  $2-2$  or  $a-a$  for  $o$ .

Corollary IV.

FROM hence, and Def. 4. we may discover the true meaning of the Expression, when it is said a *Quantity* is greater than *Infinite*, or one *Infinite* is greater than another. In these and the like Expressions  $\infty 1 = 1 + 1$

+

$+1 + 1 + 1$  &c. is always supposed the common Standard or Measure, to which all others are compared. Proper *Unity* being by *Axiom 2.* the common Measure of all Quantity, and when a Quantity  $x$  is said to be greater than *Infinite*, the meaning is, that it may be an Infinite, greater than  $\infty 1$ . and when  $\infty 2$  is said to be greater than  $\infty 1$ , it is no more than to say  $2 < 1$ . In all these *relative Infinites* admitting of Comparison, there is still a particular Finite, to which each respectively are related, and it is on these Finites that the comparing the *Infinites* among themselves is founded. Thus

$$\infty 2 = \frac{2}{0} = \frac{2}{1-1} = 2 + 2 + 2 + 2 \text{ \&c. when}$$

$$\text{compared with } \infty 1 = \frac{1}{0} = \frac{1}{1-1} = 1 + 1 +$$

$1 + 1$  &c. the finite Parts of these two Infinites, viz. 2, and 1, are the Subjects of the Comparison. And when  $\infty 2$  is said to be greater than  $\infty 1$ , it means only that the Finite's Parts, of which they are generated, are as 2 to 1, or these in the first are double of those in the second. It is the same thing as in  $\odot 2$ , compared with  $\odot 1$ , or  $2x$  with  $1x$ , nothing is meant in either, but that 2 is greater than 1 :  $\odot$ , and  $x$ , and  $\infty$ , being as to this Case equally unknown Quantities, which may be thrown

out in the Comparison, and *universally*, in all relative Infinites  $\infty n$ ,  $= n + n + n + n$  &c. and in all relative Nothings  $o n$ ,  $= n - n + n - n + n - n$  &c.  $\infty n = \frac{n}{0}$  is the Expression of the *Ratio* of the first Series, and  $o n = \frac{n}{\infty}$  is the Expression of the *Ratio* of the second Series, and these two  $\frac{n}{0}$  and  $\frac{n}{\infty}$  are proper Subjects of the Comparison, where  $n$  may admit of all the Relations that Finites have among themselves.

*Proposition V.*

*Relative Infinites, Indefinites, and Nothings* (with the proper Limitations peculiar to each) admit of all the Degrees and *arithmetical* Operations, that finite or *mathematical* Quantities are subjected to.

*Demonstrat.* This is evident from *Def.* 4, 5, and 6. and the *Scholia* adjoining to these, and is also manifest from the precedent *Prop.* and its first *Corollary*, as to relative Infinites and Nothings; and shall be afterwards shewn, as to Indefinites; to wit, that all these are still *mathematical* Quantities, capable of Increase and Diminution in *Infinitum*, and consequently must admit of all

all these *arithmetical* Operations (with proper Limitations peculiar to each) that finite Quantities are subjected to. And to confirm this, we may observe in *Nature* a Resemblance of these higher Degrees of *relative Infinites*, and consequently by *Analogy* of *relative Nothings* also. For if *Space* be *infinite*, as shall be afterwards demonstrated, it must be supposed equal to an infinite *Cube* or *Sphere*, whose *Diameter* will be as  $\infty^1$ , its Section through this *Diameter* as  $\infty^2$ , and its *Content* as  $\infty^3$ . q. e. d.

*Proposition VI.*

*Relative Infinite* has to the Finite, with which it is compared, no finite Proportion, or Finite, when compared with its proper relative Infinite, becomes relative Nothing.

*Demonstrat.* This is evident from 4 *Corollary Prop. 4.* the *Ratio* of the relative Infinites in general  $\infty n = n + n + n + n$  &c. being  $\frac{n}{a}$  that is  $\infty n = \frac{n}{0}$  therefore  $\infty n :$

$n :: 1$ . c. q. e. d. on the other Side let us suppose the *Ratio* of the relative Infinite in general  $\infty n$  to  $n$ , to be a finite Ratio  $\frac{s}{r}$  then  $\infty n = \frac{r}{s}$  which is impossible by 2 *Co-*

*rollary*



*rollary Prop. 3.* wherefore since  $\infty n : n :: 1 : 0$  by composition of Ratio's,  $\infty n + n : n :: 1 + 0 : 0$  but  $1 + 0$  is but 1. therefore  $\infty n + n$  is but  $\infty n$ , or  $n$  a finite Quantity when compared with its relative Infinite  $\infty n$  becomes 0. q. e. d.

### Corollary I.

HENCE, *relative Infinite* in general is to *Finite* in general, as *Unity* is to *relative Nothing* : or any *Finite*, when compared with *relative Infinite* in general, becomes *relative Nothing* : that is  $\infty : 0 :: 1. 0$ .

*Demonstrat.* By the *precedent Proposition*  $\infty n : n :: 1 : 0 :: a : 0 a$ , and by conversion of Ratio's  $\infty n : a :: n : 0 a$ . Supposing then  $a$  to be a *finite* Quantity, Nothing but an infinite Number of relative Nothing or 0's being equal to 1, by *Corollary 2. Prop. 2.*  $0 a$  must be still 0, or  $a - a + a - a + a - a \&c. = 0$ , by *Corollary 3. Prop. 4.* wherefore  $\infty n : a :: n : 0$ , and by composition of Ratio's  $\infty n + a : a :: n + 0 : 0$ , put  $n = 1$  and  $a = 0$  then  $\infty + 0 : 0 :: 1 + 0 : 0$  : and since  $1 + 0$  is but 1, therefore  $\infty + 0$  is but  $\infty$ ; wherefore  $\infty : 0 :: 1 : 0$ , or when any *Finite* is compared (by Addition, (or by Subtraction by Division of Ratio's)) with *relative Infinite* in general, it becomes *relative Nothing* q. e. d. The true *Analogy* is this,

$\infty n$  or  $n+n+n+\text{Ec. } a :: n : a-a+a-a+a-\text{Ec.}$  but because  $a$  is supposed Finite,  $a 0$  is the same (in Cases of Addition and Subtraction) with  $0$ . But were  $a = \infty 1$ , then  $a-a+a-a+a-a-\text{Ec.}$  would be  $\infty - \infty + \infty - \infty + \infty - \infty \text{ Ec.} = \infty \times 1 - 1 + 1 - 1 + 1 - 1 \text{ Ec.} = \infty \times a = 1$ , and  $\infty n - \infty n + \infty n - \infty n \text{ Ec.} = \infty n = n$ .

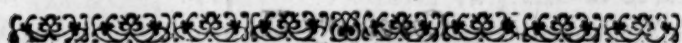
Corollary II.

Put  $n$  equal to any Integer, then  $\infty^{n-1} : n :: 1 : 0$ , but  $0^n : 0^{n-1} :: 0 : 1$ , for the first being reduced, becomes  $\infty : 1 :: 1 : 0$ , and the second being reduced, becomes  $0 : 1 :: 0 : 1$ ; and therefore when  $\infty^{n-1}$  is to be added to, or subtracted from  $\infty^n$ , it becomes  $0$ ; and when  $0^n$  is to be added to, or subtracted from  $0^{n-1}$  it becomes also  $0$ , by *Scholium Def. 6.* and the Case is the same when the inferior Powers suppose  $\infty^{n-2}$ , or  $\infty^{n-3} \text{ Ec.}$  are to be added to, or subtracted from  $\infty^n$ , or when  $0^n$  is to be added to, or subtracted from the inferior Powers  $0^{n-2}$  or  $0^{n-3} \text{ Ec.}$  in all such Cases it is evident from the precedent *Prop.* and its 1. *Corollary*, that  $\infty^{n-2}$   $\infty^{n-3}$  and  $0^n$  become  $0$ .

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*Scholium.*

FROM these *Propositions* and *Corollaries*, an *Arithmetick* of Infinites may be drawn out, different from any hitherto published, of no contemptible Use, or narrow Extent in *Algebra*, and *Geometry*; as will be in some Measure shewn by the third Chapter.



*The Arithmeticks of Infinites.*

**A**ddition.  $2\infty + 3\infty = 5\infty$ .  $a\infty + 2a\infty = 3a\infty$ .  $\infty^2 + 2\infty^2 = 3\infty^2$ .  
 $\infty^n + p\infty^n = 1 + p\infty^n$ .  $a\infty + e\infty = a + e\infty$ .  $3\infty + 10 = 3\infty$ .  $a\infty + a = a\infty$ .  $\infty^2 + \infty = \infty^2$ .  $\infty^n + \infty^{n-1} = \infty^n$ .  
 Let  $q$  be less than  $n$ , by a finite Integer, then  $\infty^n + \infty^{n-q} = \infty^n$ .

Subtraction.  $5\infty - 2\infty = 3\infty$ .  $10a\infty - 3a\infty = 7a\infty$ .  $6n\infty - 4n\infty = 2n\infty$ .  $5\infty - 7\infty = -2\infty$ .  $7\infty^3 - 5\infty^3 = 2\infty^3$ .  $9\infty^p - 5\infty^p = 4\infty^p$ .  $5\infty^p - a\infty^p = 5 - a\infty^p$ .  $3\infty - 7 = 3\infty$ .  $a\infty - a = a\infty$ .  $\infty^p - \infty^{p-1} = \infty^p$ .  
 Let  $q$  be an Integer, less than  $p$ , then  $\infty^p - \infty^{p-q} = \infty^p$ .

Multiplication.  $\infty \times 3 = 3\infty$ .  $\infty \times a = a\infty$ .  $\infty \times \infty = \infty^2$ .  $\infty^2 \times \infty^3 = \infty^5$ .  $\infty^p \times \infty^q = \infty^{p+q}$ .  $a\infty \times e\infty = ae\infty^2$ .  $3\infty \times a\infty = 3a\infty^2$ .

$$a \times 4 \infty x = 12 \infty^2 a x. - 2 \infty x - 3 \infty$$

$$= 6 \infty^2. - \infty a x + \infty = - \infty^2 a.$$

Division,  $\frac{\infty}{2} = \frac{1}{2} \infty$ , or  $\frac{\infty}{2} = \frac{1}{2} \infty a$

$$a^2 \infty = 2 \cdot \frac{3 \infty}{4 \infty} = \frac{3}{4} \frac{\infty a}{\infty e} = \frac{a \infty^2}{e \infty} = \infty.$$

$$\frac{\infty^3}{\infty} = \infty^2. \frac{\infty^p}{\infty^q} = \infty^{p-q} + \frac{3 \infty}{- \infty} = -3 \cdot \frac{4}{2}$$

$$\infty = +2 \cdot \frac{\infty a}{\infty e} = \frac{a}{e}$$

Fractions,  $\infty^{\frac{2}{3}} + \infty^{\frac{4}{5}} = \infty^{\frac{22}{15}} \infty^{\frac{a}{e}}$

$$\infty^{\frac{p}{q}} = \infty^{\frac{a q + e p}{e q}} = \infty^{\frac{3}{2}} + \infty^{\frac{4}{5}} = \frac{2}{5} \infty.$$

$$\infty^{\frac{a}{e}} - \infty^{\frac{p}{q}} = \infty^{\frac{a q - e p}{e q}} \infty^{\frac{a}{e}} \times \infty^{\frac{p}{q}} =$$

$$\infty^{\frac{ap}{eq} - \frac{a}{e}} \infty^{\frac{p}{q}} \left( = \frac{e p}{q a} \infty^{\frac{5}{4}} \right) \infty^{\frac{2}{3}}$$

$$\left( = \frac{8}{15} \right)$$

Those who are ever so little acquainted with the *specious Arithmetick*, will easily understand the Reason and Truth of these Operations.

*Proposition VII.*

*Indefinite Quantities* are not properly either Finite or Infinite, but between both.

*Demonstrat.* An indefinite Quantity is some mean Proportional, between finite and relative

lative Infinite, *per Def. 6.* that is, one of those Infinites will be  $\infty = \sqrt{\infty}$  & put  $\infty = 1$ , and then  $\sqrt{\infty}$ , will be an indefinite Number, now this  $\sqrt{\infty}$  can neither be Finite (and this Manner of Reasoning will hold good of any other Indefinite whatsoever) nor Indefinite; not Finite, else  $\infty$  would be Infinite, which is impossible; nor Infinite, for for the least possible Infinite must be Infinite in general, divided by the greatest possible Finite  $x$ , and then if  $\sqrt{\infty}$  be infinite,  $\sqrt{\infty} = \frac{\infty}{x}$  and  $x^2 = \frac{\infty}{\infty} = \infty$ , which is also absurd. Again if  $\sqrt{\infty}$  be infinite, then is  $\infty \times \frac{1}{\sqrt{\infty}} (= \sqrt{\infty}) = \text{infinite}$ , and  $\frac{1}{\sqrt{\infty}} =$  to its relative infinitely small Part, or its relative Nothing  $= 0$ , and so  $\infty \times 0 = \infty \times \frac{1}{\sqrt{\infty}} = (\sqrt{\infty} =)$  to infinite. But by *Corollary 2. Prop. 2.*  $\infty \times 0 = 1$ , and instead of  $\infty \times 0$ , putting its value 1. in this last Equation, it will be  $\infty \times 0 = 1 = \infty \times \frac{1}{\sqrt{\infty}} = \text{Infinite}$ , by Supposition; and this supposed Infinite would become 1, which is absurd. q. e. d.

Scho-



*Scholium.*

TAKING the Instance of the indefinite Quantity proposed  $\sqrt[\infty]{1} = \sqrt{1+1+1+1} \&c.$  It's plain the Root of any given Number grows greater or less, as the Number expressing or denominating the Root is less or greater; and in  $\sqrt[\infty]{1}$ , its infinitely little Root or  $\sqrt[\infty]{1}$  may be any finite Number greater than Unity, as has been already shewn; and its infinitely great Root or  $\sqrt[\infty]{1}$  is infinite. And between these lie all the Indefinites that can be formed on 1 and  $\infty$ ; to wit  $\sqrt[2]{\infty}$ ,  $\sqrt[3]{\infty}$ ,  $\sqrt[4]{\infty}$ , &c. and none of these can be properly called either Finite or Infinite, but are in a perpetual Gradation towards either of these Extremes, as the Number that denominates the Root grows greater or less; and they never become actually finite, but when the Number denominating the Root is actually infinite, nor actually infinite, but when the Number denominating is Unity. And between these two Limits, they are neither actually finite nor infinite. Next to  $\sqrt[1]{\infty} = \infty$  (in order of the simplest Indefinites,) is  $\sqrt[2]{\infty} = 1 + \frac{1}{2} + \frac{3}{\infty} + \frac{5}{16} + \frac{35}{128} \&c.$  where- in (by the quick Increase and Greatness of the succeeding Terms) the last becomes  $\frac{\infty}{\infty}$  in a few Number, so to speak, of these Terms,

Terms, where the Series terminates, and so their Sum  ${}^2\sqrt{\infty}$ . becomes thereby less than  $\infty$  1. as will be more fully explain'd in the following Chapter, and in  ${}^3\sqrt{\infty}$  (for example)  $= 1 + \frac{1}{3} + \frac{6}{3^2} + \frac{6^2}{3^3} \&c.$  we see the Terms of the Series converge yet faster, and will thereby terminate at  $\frac{1}{3}$  much sooner. And so their Sum  ${}^3\sqrt{\infty}$  will be much less than  ${}^2\sqrt{\infty}$ . And for these (even when they rise to fractional Exponents, whose Numerators are greater than 1, such as  $\infty^{\frac{1}{2}}$ ,  $\infty^{\frac{3}{4}}$ ,  $\infty^{\frac{5}{6}}$  &c.) an *Arithmetick* may be formed, according to the Example laid down in the *Scholium* of the precedent Proposition, with this ad-

dition, that an Indefinite as  $\infty^{\frac{q}{r}}$  or  $p^{\frac{q}{r}}$  multiplied by another Indefinite  $\infty^{\frac{r}{s}}$  gives the Product  $\infty^{\frac{q}{r} + \frac{r}{s}}$  which becomes Infinite, when  $qs + pr$  is  $=$  or  $< ps$  and Indefinite when  $qs - pr$  is  $=$  or  $> ps$ .

And if an Indefinite as  $\infty^{\frac{q}{r}}$  be divided by an Indefinite, as  $\infty^{\frac{r}{s}}$  the Quotient  $\infty^{\frac{q}{r} - \frac{r}{s}}$  is Infinite, when  $qs - pr$  is  $=$  or  $< ps$ , but is Finite when  $qs - pr = 0$  and only Indefinite, when  $qs - pr > ps$ . as

it ought to be according to the Arithmetick of Surds.

*N. B.* That in expressing an Indefinite by  $\infty p$  it is always supposed that the Number  $q$  is less than  $p$ , for if  $q$  be either greater or equal to  $p \times$ , then  $\infty p$  tho' it may be an Indefinite of the superior Degrees, yet it is always infinite in its Value.

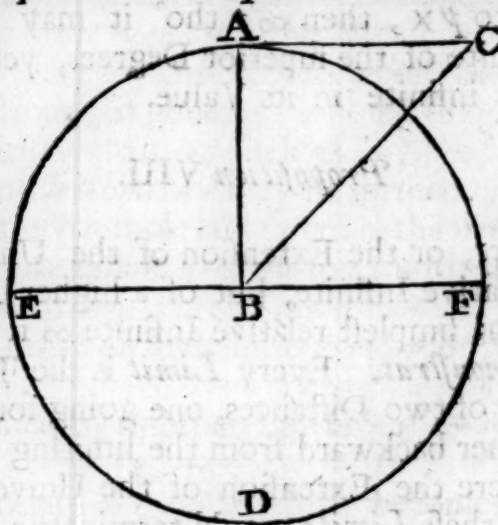
### Proposition VIII.

*Space*, or the Extension of the Universe, is a relative Infinite, but of a higher Degree than the simplest relative Infinite  $\infty 1$ .

*Demonstrat.* Every *Limit* is the Termination of two Distances, one going forward, the other backward from the limiting Point, and were the Extension of the Universe limited, these *Limits* wou'd terminate a Space, beyond these *Limits*, as well as within them. That is, either the universal *Space* must be unlimited, and consequently infinite, or there must be *Space* beyond the Limits of universal Space, which is absurd. Again, if the Extension of the Universe were limited any way, so as to become finite, then a *Sphere* of a finite *Diameter* might be found equal to it. For the *Cube* of a finite Side may be found equal to any finite *Content* whatsoever.

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ver, as is well known, and the *Radius* of a *Sphere* equal to this *Cube* is the *Product* of the *Side* of the *Cube*, multiplied into the *cube Root* of  $\frac{3}{2}$  *Parts* of the *Ratio* of the *Radius* to the *Circumference*. Let us suppose the whole finite *Extension* of the *Universe* equal to the *Sphere* whose *Radius* is



*AB.* Let this *Sphere* be cut by a *Plane* through its *Center*, and the *Section* be the *Circle ADFE*, it is certain from the *Elements of Euclid*, that to any given Point *A*, a *Tangent AC* may be drawn, of which only the Point *A* falls upon the *Circle*; the rest of the Line *AC* falling without it. From whence it is evident, that there must be *Extension* without this *Circle*, or the *Sphere*

*Sphere* by the Section of which it is generated. Since a *Plane* passing through this *Tangent*, and perpendicular to the *Plane* of this *Circle*, will only touch the *Sphere* in a Point; by which there will be an extended Distance remaining between the *Circumference* of the *Sphere*, and the *touching Plane*, in all their Points excepting that one at *A*; And since this is true of every *assignable* Extension how great soever, less than Infinite. It is evident the Extension of the Universe is greater than any *assignable* Extension, how great soever, that is, by *Axiom* 1. it is *relatively Infinite*, which is still more evident from *Def.* 4. Since its Parts are finite and their Sum only infinite, and seeing this Infinite is of three Dimensions, or as  $\infty 1$  in Height, Breadth, and Depth, therefore it must in its Content be as  $\infty^3$ , that is, of a superior Degree to  $\infty 1$ . q. e. d.





## CHAP. II.

*Of the PHILOSOPHICAL Principles of reveal'd Religion.*

## • Lemma I.

THERE is in all the Works of Nature a *Symmetry* and *Harmony*, running on in a perpetual *Analogy* (with proper Limitations arising from the different Circumstances of the several Parts) through the whole and the Parts; or there is a *regular Connexion*, and *uniform Proportion* between similar Causes and Effects, a Congruity between the End and the Means. An Aptitude between the Faculty and its Acts, and between the Organs and their intended Uses in the whole, and in the several Parts of this *System* of Things.

*Demonstrat.* This is evident from innumerable Instances already discover'd and ascertain'd. And every new Discovery in the most minute Part in the Works of Nature carries along with it a fresh *Demonstration* of this Proposition; one must be intirely ignorant of *Philosophy*, and *Mathematicks*, to want a Cloud of Witnesses to this Truth. For Instance, the *sesquialter Proportion*, of the

the periodical Times of the Revolutions of the *primary Planets* about the *Sun* ; and of the *secundary Planets* about the *primary Ones* ; to their *middle Distances* from the *Sun* and *primary Planets* respectively, obtains universally. Their *Magnitudes*, *Gravities*, *Densities*, and their *Velocities* in their *Orbits*, in respect to, and about the *Sun* and the *primary Planets*, in the *Planets*, *Comets*, and *Satellites*, are in a regular and comely Proportion ; the same *Gravity*, the same Law thereof, and the similar Effects of both, obtains through the whole material *System* of Things. The *Reflexions*, *Inflexions*, and *Refractions* of Light, are the same in all the *planetary* and *cometary* Bodies and Regions ; as they are on our terrestrial Globe, with due Regard to the different *Densities* of the *Mediums*. The *Circulation* of the *Fluids*, the *Manner* and *Organs* of *Respiration* and *Generation*, are *analogically* the same in *Man*, *Brutes*, and *Vegetables* ; with proper Limitations arising from the differing Circumstances of these Gradations of Animals. The *general Laws* of *Fluids*, *Elasticity* and *Gravity*, obtain in *animal* and *inanimate Tubes*, but so far as they are alter'd in the first by *collateral Causes*. The whole of *Philosophy* and *Mathematicks* is nothing but particular Instances of this beautiful *Analogy*, and the preceeding Chap-

ters of the *first Part* contain nothing but particular Instances thereof. And if we descend into the *spiritual World*, we shall find this beautiful *Analogy* preserv'd, as far as the different Circumstances of these Beings will permit. And if in this Demonstration, it were allow'd to take in the Supposition of a *Being infinitely perfect*, who contrived and executed the whole, and the several Parts of this *System* of Things; it is impossible it shou'd be otherwise: A Being infinitely Wise, Simple, and One, must necessarily bring about similar Ends by similar Means, and perform all his Works the plainest, most simple and shortest Way possible; due Regard being had to the whole, and the different Circumstances of the several Parts. *Wisdom in Things*, is their *Symmetry*, *Regularity*, and *Aptitude* for obtaining their design'd Ends and Purposes. The *Wisdom* of a *Machine* consists in the due proportioning of the several Parts to one another, and to the whole, for obtaining its proposed End. *Disproportion*, *Irregularity*, *Discord*, and the having no View or Design, are the surest Proofs, and Indications of Chance, Impotence and Folly: A wise Man performs all his Works in Number, Weight and Measure, and sure *Infinite Wisdom*, *Simplicity*, and *Unity*, must accomplish all its Works, with the most consummate *Harmony*, *Proportion*,  
and

and *Regularity*. And this in the following Parts of this Treatise, for Brevities sake we shall call the ANALOGY OF THINGS.

Lemma II.

THIS ANALOGY OF THINGS duly instituted, is as certain a *Demonstration* of the *Existence* and *Wisdom* of the Author of these Things, and of the Contriver of this *Analogy*, as also of the true Nature and Qualities of these Things discovered by this *Analogy*, as any *mathematical Demonstration* is of the Proposition proposed.

*Demonstrat.* No Effect can be, without its proper Cause; a wise and regular Effect must be produced by a wise and intelligent Cause, and an *infinitely wise*, and *infinitely complicated* Effect, must necessarily imply an *infinitely wise* and *Omniscient* Cause. These are so certain and infallible *Axioms*, that I know not if in all the Compass of human Knowledge, any others come up to the same Degree of Evidence: And he can be no proper Subject of *Philosophy* or *Mathematicks*, who cou'd seriously deny them, since the whole Evidence of both these Sciences suppose the first of the *Axioms*, and the rest are but like multiplying both Sides of the same *Equation* by the same Terms. I have already observed, that the *Wisdom* of

an *Effect*, or System of Effects, consists in the Proportion or *Analogy* of the several Parts to the whole and to each other, and of the whole, to the End propos'd ; and that Irregularity and no Proportion is the surest Evidence of Want of Contrivance, Wisdom and Design. Now since the ANALOGY OF THINGS, just now demonstrated to be found in all the Works of Nature, in the whole, in every the most minute Part : And in these Instances of this *Analogy*, without Number, and without End. (Every new Step in the Knowledge of Nature discovering fresh Instances of this *Analogy*) all these, I say, do necessarily infer an *Infinity of infinitely wise* Effects ; and therefore these Effects must as necessary infer the Existence of the Author of these Effects, and the Wisdom of the Contriver of this *Analogy*, as an Effect infers its Cause ; that is, as certainly as any *mathematical* Demonstration infers its Proposition ; since its Certainty depends on the Connexion between Causes and Effects, and the Truth of this *Analogy* in general. I say in the next Place, that the true Nature and Qualities of these Effects or Things, discovered by this *Analogy*, duly instituted, may be as certainly concluded from this *Analogy*, as any *mathematical* Demonstration concludes its Proposition. *Mathematicks* and *Philosophy*,



*sophy*, so far as they are just and genuine, are but Branches of this *Analogy*. *Mathematics* are but this *Analogy* apply'd to *Figures* and *Numbers*. *Philosophy*, properly so call'd, is but this *Analogy* apply'd to *Bodies*, or *Systems* of these; or to the abstracted Natures of Things. Both suppose the Truth and Necessity of this *Analogy*, without which they are but *Fargon* and *Romance*. An Instance or two will make the whole Matter clear. Suppose it were required to find the *refracted Ray*, when the *refractive Powers* of the two *Mediums*, with the Inclination of the *incident Ray*, on the *incident Plane*, are given. Let us borrow a *Corollary* from this *Analogy* of Things, to wit, that the Distance between any fix'd Point in the *Incident*, and another in the *refracted Ray*, (the *refractive Powers* of the two *Mediums* being regarded,) is the shortest possible; and with this *Corollary* make an exact Computation; we shall then find the *refracted Ray* precisely the same with that found out, from other different Principles and Methods (suppose of *Trigonometry*) where this *Corollary* has had no Place, as the *Geometers* have shown. This *physical Demonstration* of this particular Property of Light, (to wit, that in all *Incidences*, the *Sine* of the *Angle* of *Incidence*, is to that of *Refraction*, in a constant *Ratio*)

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*Ratio*) arising from this *Analogy*, is as certain a Proof of the Existence and Wisdom of the *Contriver* of this *Analogy*, as also of the true Nature of Light, (as to this particular Property) as any *mathematical* Demonstration (suppose that by *Trigonometry*) is of the true Nature of the *incident* and *refracted Ray*. For who but an *Omniscient Artificer* cou'd contrive Light so? That among all the infinitely different possible Ways, between two assign'd Points, it shou'd single out that one, which is the shortest? And it is certain that the *refracted Ray* is as truly found out by Virtue of this *Analogy*, as by any other more *geometrical* Method. Another Instance may be taken from the regular and harmonious Progressions of *infinite Series's*; for Example, in the Powers of the *Binominal Root*,  $(a+y)^n = a^n + n a^{n-1} y + n \times \frac{n-1}{2} a^{n-2} y^2 + n \times \frac{n-1}{2} \times \frac{n-2}{3} a^{n-3} y^3$  &c. or the *Series* produced by Division  $\frac{1}{1-n} = 1 + n + n^2 + n^3 + n^4$  &c. In

a Word, every particular *Problem* in *Algebra* and *Geometry* might be brought as Instances of this beautiful *Analogy* of Things; and those who are conversant in the more abstruse Speculations of *abstract Geometry*, can furnish themselves with Instances so surprising and extraordinary, of the Constancy of

of Nature in this beautiful Harmony, and comely Proportion, even in the largest Computations and most complex Constructions, as far surpass these Instances I have brought. Now what but an infinitely wise Being cou'd have constituted the *intellectual Species* of Things so admirably? That all the Terms of these Progressions shou'd thus go on in such regular and harmonious Proportions, that every succeeding Term shou'd be made up of the preceeding ones, modify'd after one constant Way, that by the Cast of an Eye, the said succeeding Term shou'd be assign'd? Whereas they might have been ordered, other infinitely different Ways, so as to have afforded no regular Progression. And does not this *Analogy* and harmonious Progression of these *Series's* as certainly give the succeeding Terms, as the actual Operations of *Algebra* do? Many more, and yet infinitely more surprizing Instances of this beautiful *Analogy*, and of the Inferences drawn from those assign'd, might be given. But these may suffice for an Illustration of this *Lemma*.

*Lemma III.*

THE Rules which seem naturally to arise, for the due Institution of this *Analogy* of Things, may be reduced to these three, 1.  
That

That the Quality, Property, or Idea, on which the *Analogy* is instituted, be as simple an one, as possibly may be; and intirely the same, both in the known and unknown Subjects of the *Analogy*. Thus when we reason from the Nature and Properties of *Light* and *Gravity* on our *Earth*, to the Nature of *Light* and *Gravity* in the *Planets* and *Comets*. We must separate these into their most simple *Ideas* and *Properties* and (to avoid Confusion) as much as may be, and institute an *Analogy* for each, to discover if the *Analogy* hold good in the Complex, and precisely keep to the same *Properties* in the *Planets* and *Comets* the unknown Subject of the *Analogy*, as were supposed in the *Earth*, the known Subject of the *Analogy*. Else we shall run into Confusion and *Paralogism*. 2. The necessary Limitations arising from the different Circumstances of the two Subjects of the *Analogy*, as far as they may be known, must be cautiously and carefully taken into the State of the Question. Thus in the first Instance, assign'd in the second *Lemma*, if the different *refractive Powers* of the two *Mediums* had not been precisely entered into the *Equation*, the Conclusion must have prov'd different from that found out by other Methods. Thus also in reasoning from the Manner of *Generation* in *Animals* to that of *Vegetables*. If the Limitations

imitations arising from the different Circumstances and Orders of Being, in these two Subjects of the *Analogy*, be not carefully taken into the Reasoning, the Conclusions must prove false and erroneous. 3. Both the Subjects of the *Analogy* must be known and examined into, as far as may be, in regard to the other Qualities different from those, the *Analogy* is instituted upon: But especially the known Subject of the *Analogy* must be as fully known as is possible, in regard to those Properties on which the *Analogy* is instituted. Thus the more fully we understand the Nature and Qualities of our *Globe*, especially as to *Light* and *Gravity*, and of the *Comets* and *Planets* as to their other Qualities, the more full and perfect shall our Conclusions be in regard to these assign'd Qualities, in the *Planets* and *Comets*: The more fully we understand the Circulation of the Fluids in *Animals*, the more aptly shall we apply them to *Vegetables*. These Rules duly observed will render the Conclusions drawn by Virtue of this *Analogy of Things*, as certain as any *mathematical* Demonstration whatsoever.

*Proposition IX.*

THE *visible, intellectual, and created Species* of Things, are Pictures, Images, and Representations of the *invisible, architypal*,  
and



and *increated Species* of Things in the Mind of the supreme Being.

*Demonstrat.* This is evident from *Lemma* 1 and 2. For since it is certain, that there are to be found in all the Works of Nature a beautiful Harmony, a comely Proportion, and an exact Symmetry running thro' the whole. And since this *Analogy of Things*, necessarily infers the Existence of the Author of these Things, and the Wisdom of the Contriver of this *Analogy*. These Things and this *Analogy* cou'd come from nothing else but from their original *Ideas* and *architypal Paterns*, in the *divine Mind* or *Imagination*, and their Harmony and Proportion can possibly arise from nothing but their being Representations of his *Ideas*, who is *Omniscient*, and does every Thing in *Number, Weight* and *Measure*. There being no other possible Way they cou'd be contrived; and he being *Supreme* and *One*, cou'd find nothing without himself that they should represent. Besides, it is absolutely impossible, that infinite Power and Perfection shou'd bring any Thing into Being, that had not his own *Signature*, Stamp, or Image on it, for there cou'd be nothing besides *himself*, whose Images they shou'd be; and it is absurd to imagine they shou'd represent nothing at all. Therefore of necessity they must be Pictures, Images, and Representations

tions of their *Ideas* and original Paterns in the Mind of the supreme Being. q. e. d.

Corollary I.

HENCE it is evident that the *visible, intellectual, and created Species* of Things, are Images, Pictures, and Representations of the *divine Attributes*, more or less perfect, according to their Order in the Rank of Beings. For since they are Images of the *architypal Ideas* in the *divine Mind*, or *Imagination*, and since there was nothing without him, to beget these *Ideas*, and nothing but his own infinite Perfections, that cou'd represent themselves to him, in order to make these *Ideas* arise in the Mind. Therefore of necessity, they must represent his *Attributes* or *Perfections*, the only Thing he cou'd have to copy out.

Corollary II.

HENCE it is evident, that with proper Limitations arising from the *infinitely-infinite* Distance ; (so to speak) between Finite and *absolutely Infinite* we may reason *analogically* from the Nature and Attribute of the *Supreme intelligent Being*, to the Nature and Properties of *finite intelligent Beings* ; and from these to those ; to wit, by supposing

sing these last Images, Pictures, and Representations at an infinite Distance of the first.

*Proposition X.*

A *mathematical* Point, and infinite Space are the two *Limits* of natural or created Things, as to Quantity or Extension, in Ascending or descending from Finite. And neither of them is any Part or Multiple of the other.

*Demonstrat.* This is evident from *Prop.* 8. for natural or material Things can have no more Dimensions than three, and therefore the biggest Extreme in created Things, can rise no further by the *Analogy* of Things than to  $\infty$  raised to the third Dimension. And since infinite Space is as  $\infty^3$ , therefore infinite Space is the biggest Extreme, or Limit of natural Things ascending from Finite. And since a *mathematical* Point is the Surface (so to speak) of the Extremity of a *mathematical* Line, which has but one Dimension, it must of Necessity be the least Extreme in descending from Finite: In a Word, we can rise no higher in natural and material Things than infinite Space, since that is the *Locus Universalis* of all created Beings. And we can descend no lower than a *mathematical* Point, since every Thing below it (if possible) cou'd have no Place at all, or  
wou'd

wou'd be no where, that is, wou'd be absolute Nothing. And neither of them can be any Part or Multiple of the other, since no Number of Points, no not even an infinite Number can make any real natural Quantity, not so much as a *mathematical* Line, therefore &c. q. e. d.

Corollary I.

HENCE a *mathematical Point* and *universal Space* are true and genuine Opposites in Nature, and in the *Analogy* of Things, and between these two lie all created finite Subsistences. No natural Thing can be bigger than *infinite Space*, and no natural Thing can be less than a *mathematical Point*, they differ the most widely that natural Things possibly can, and have nothing common but *Entity*, and so are true and genuine Opposites.

Corollary II.

HENCE in the *Analogy* of Things, *Matter* cannot be infinite, in any Sense of Infinite ascending from Finite. For since *Matter* is of those natural Things, that necessarily require all the three Dimensions, were it infinite, by the *Analogy* of Things, it wou'd necessarily be as  $\infty^3$ , that is, it wou'd be equal to infinite Space; but since the Necessity of a *Vacuum* has been demonstrated in the first Chapter of the first Part, it cannot be

PART II.

E

equal

equal to infinite Space, and since it cannot be infinite any wise but as infinite Space, by the *Analogy* of Things. Therefore *Matter* cannot be infinite, in any Sense of Infinite ascending from Finite.

*Proposition II.*

*Absolute Nothing*, upon a real Quantity produces no Effect at all. Or *absolute Nothing* can be susceptible of none of these *arithmetical* Applications, to which real Quantities are subjected.

*Demonstrat.* Suppose absolute Nothing upon a real Quantity cou'd produce any Effect. Let the real Quantity be  $a$  and the Effect  $m$ , then absolute Nothing  $\times a = m$  that is, absolute Nothing is equal to  $\frac{m}{a}$ , that

is, absolute Nothing is a real Quantity, as  $\frac{m}{a}$  is, which is absurd. Now since absolute

Nothing multiplied into a real Quantity can produce no Effect, for the Reason and by the Argument now assign'd, neither can it when divided by a real Quantity, by the same Way of reasoning, since Division by  $a$  is the same with Multiplication by  $\frac{1}{a}$ . And

since Multiplication and Division are the same with Addition and Subtraction reiterated



ted as often as the Multiplier and Divi-  
for implies : Therefore *absolute Nothing* is  
susceptible of none of those *arithmetical*  
Applications that real Quantities are subjec-  
ted to. q. e. d.

*Corollary.*

*Absolute Nothing* therefore, when apply'd  
to real Quantities by Multiplication and Di-  
vision (and consequently, when by Addition  
and Subtraction) implies, that there is nei-  
ther Product nor Quotient, *that is*, that the  
real Quantity is neither multiplied nor divi-  
ded, but remains unaltered.

*Proposition XII.*

In the *Analogy* of Things ; as a *mathe-  
matical Point* is to *universal Space*, so is  
*absolute Nothing* to the *absolute and supreme*  
*Infinite*.

*Demonstrat.* By *Prop. 10.* a *mathemati-  
cal Point*, and *universal Space*, are the two  
*Limits* of material or natural Things ; they  
involve simple, clear and distinct Ideas, and  
are as well known, as any of the Objects of  
human Knowledge. *Absolute Nothing* is  
one of the Limits of the *universitas rerum*  
*omnium*, to wit, that in descending, below  
which nothing can fall ; and therefore by

the Rules laid down, for the *Analogy* of Things, the *absolute* or *supreme Infinite* must be the other Limit, beyond which nothing can rise in ascending, and between these two, all *Subsistences*, Finite, Indefinite, and relative Infinite are concluded. Wherefore by *Lemma* 3. and *Prop.* 9. as a *mathematical Point*, &c. q. e. d.

### Corollary I.

HENCE *absolute Infinite* can neither be encreas'd nor diminish'd. For *absolute Infinite* and *absolute Nothing* being the *Limits* of all Things whatsoever, *absolute Infinite* must be the greatest of all Things, and so can neither be encreas'd nor diminished, else it cou'd neither be the *greatest*, nor the ascending *Limit*.

### Corollary II.

HENCE *absolute Infinite* is *One*, and *Individual*. Because being neither capable of Encrease nor Diminution, and being the ascending *Limit*, it must be one: And individual, because there can be no other like it, it being the one ascending *Limit*.

Corol

## Corollary III.

HENCE *absolute Infinitude* is only compatible to the divine Nature, and to nothing else. He being the greatest of all Beings, one, and *individual*, or the single possible Being of the same Nature.

## Corollary IV.

*Universal Space* is the Image and Representation in Nature, of the *divine Infinitude*, for since by *Prop. 9.* the created *Species* of Things are Images of the increated, and by *Prop. 10.* *universal Space* is the greatest Limit of the visible Creation, or material System of Things. Therefore *universal Space* is the natural Image of the greatest Limit of all Things, or of the *universitas rerum omnium*. That is, by *Prop. 12.* and the preceding Corollary, *universal Space* is the natural Image of the *divine Infinitude*.

## Corollary V.

HENCE *universal Space* may be very aptly called the *Sensorium Divinitatis*, since it is the Place wherein all natural Things, or the whole System of material and compounded Beings, are presented to the *divine Omnis-*

ence. Infinite Space is the Image of the *divine Infinitude*, wherein as in a Picture of him (in whom all Things live, and move, and have their being) all created Things present and manifest themselves to the intuitive View of the *supreme Infinite*, and therefore in the *Analogy* of Things, infinite Space is to the *supreme Infinite*; what a human *Sensorium* may be supposed to be to Men.

### Corollary VI.

AN *absolute infinite Creature* is a Contradiction, because *absolute Infinite* is *one* and *individual*, by Corollary 2. of this.

### Scholium.

ONE very remarkable Difference between *Finite, relative* and *absolute Infinite* is this. *Finite* may still be increas'd by it self, and yet continues its finite Nature, 'till the Number of Additions be actually Infinite. *Relative Infinite* may be increased, not by Finite, but by *it self*, and still continues its Nature, after *infinitely-infinite* Additions, without Bounds or Limits: For let it be ever so often increas'd by it self, it continues but *relative Infinite*. But *absolute Infinite* can be increas'd by nothing, not even by it self, else it cou'd not be the absolute or great-

greatest Infinite. In natural Things *Finite*, and *relative Infinite* never rise higher than the third Power. The *Comets* and *Planets* move in *Orbits*, that are Curves of the *second Power*. *Projectiles* move in *parabolic Lines*, which are of the same Order. The *Curvature* of the *Surface* of all the *celestial Bodies* are of the *conical-section-kind*. The *Surfaces* of Fluids, rising in great or small Channels, are of the same Rank. Nature admits but of three Dimensions, and I know of no natural Effect that rises higher than the third Power. Even the *irrational Curves* that Nature forms, in conducting *solid Bodies*, and the *Rays of Light* (thro' different *refracting Mediums*) the shortest and easiest Way: And in some other of her Operations, such as the *Cycloid*, *Conchoid*, *Catenaria*, *Velaria*, *Curva Elasticitatis*, *Logarithmica*, the *Spiral* and the like, all of them are of the lowest Rank of their Order: And their Natures imply only Portions of *Curves*, or of their *Areas*, of the *conical-section-kind*, the lowest Order of all *Curves* whatsoever. So true it is that Nature perpetually brings about her Purposes *the shortest and simplest Way*, and keeps constant to this beautiful *Analogy* of Things. But since Quantity may be still increased,  $\infty$  may be supposed to rise to higher Powers than  $\infty^3$  in the intellectual *Species* of Things, such as



are,  $\infty^4$ ,  $\infty^5$ ,  $\infty^\infty$ ,  $\infty^{\infty\infty}$  &c. Now tho' these Powers of  $\infty$  superior to  $\infty^3$ , can have no Place in natural Things, or the material *System*, yet are they the *intellectual Species* of created Things, and are in their respective Orders, *Images*, and Representations of their *architypal Ideas*, in the divine Mind, and Instances of the unexhaustible Store of manifold Wisdom in the divine Nature. And as *infinite Extension* is the Image in Nature of *absolute Infinite*, so  $\infty^{\infty\infty}$  &c. is the Image of the same absolute Infinite, in the *intellectual Species* of Things. And by the *Analogy* of Things, relative Nothing, and relative Infinite are the *intellectual Images* of *absolute Nothing*, and *absolute Infinite*, so universally does this *Analogy* hold good.

*Proposition XIII.*

*Absolute Infinite*, in the *Analogy* of Things, is the precise and proper Opposite to *absolute Nothing*.

*Demonstrat.* This is plain from *Prop. 12.* and its *first Corollary*. For since a *mathematical Point* is the precise and proper Opposite to universal Space; and since in the *Analogy* of Things, a *mathematical Point* is to *infinite Space*, as *absolute Nothing* is to *absolute Infinite*: Therefore *absolute Infinite*

nite is the precise and proper Opposite to *absolute Nothing*. Besides, *absolute Nothing* and *absolute Infinite*, being the *Limits* of the *Universitas rerum omnium*, in ascending or descending from created finite Beings. They must be precise and proper Opposites, having nothing common but their being *Entia and Limits*. q. e. d.

Corollary I.

SINCE *absolute Nothing*, in its positive *Idea*, implies the utmost impossibility of, and the most extreme Contradiction to *Reality* or real Being, as it most certainly does, for it is not possible to conceive a greater Contradiction to, or Impossibility of *Reality* or real Being, than is imply'd in the positive *Idea* of *absolute Nothing* or *Non-entity*, beyond which there is no further descending from *Reality* or Being. (It having no real *Entity*, tho' it may be class'd, as to its negative Conception, in the *Category* of *Entia rationis*.) Therefore by the *Analogy of Things*, *absolute Infinite* must imply in its positive *Idea*, and that necessarily, the utmost Possibility, and the most extreme Necessity of *Reality* or real Being

Corol-

## Corollary II.

WHEREFORE since *absolute Nothing*, in its positive *Idea*, implies the utmost Contradiction to real Being, since *absolute Infinite* is the precise Opposite to *absolute Nothing*, since also being Opposite in every Thing (except as *Entia* and *Limits*, which do not effect the Realities in *absolute Infinitude*, as to its positive *Idea*) *absolute Infinitude* must imply in its positive *Idea* the utmost Necessity of *Reality*, or real Being; therefore of Necessity the *absolute Infinite* must really *Be*, or the Being to which *absolute Infinitude* is only compatible, must necessarily exist.

## Proposition XIV.

*Finitude* and *Infinitude*, when apply'd to natural and created Things, in their positive *Ideas*, imply not *Realities*, but the *Modes* of *Realities*.

*Demonstrat.* *Finitude*, when apply'd to natural or created Things, imports only in it's positive *Idea* the Proportions of the several Degrees of *Affections*, or *Properties* of these Things to one another. *Infinitude*, the unboundedness of these Degrees of *Affections*, or *Properties*. *Finitude* and *Infinitude*,

tude, in themselves, abstracted from a proper Subject or *Substantive*, are incomplete *Ideas* in natural or created Things. *Infinite Extension, Number, Duration, Wisdom, Knowledge, &c.* are complete *Ideas*, whereof these *Realities* are the Subjects or *Substantives*, and the *Infinity*, the *Epithet* or *Adjective*: Wherefore *Finitude* and *Infinity* in natural or created Things, being but *Adjuncts* to *Realities*, in their positive *Ideas*, do not imply *Realities*, but the *Modes* of *Realities*. q. e. d.

Corollary I.

HENCE the *Modes* of natural or created Things are *Realities*, in the supreme or *absolute Infinite*, or the *Modes* of natural or created Things, when *analogically* elevated to the supreme, and *absolute Infinite*, are in him not *Modes*, but the utmost *Realities*. This is plain from the *preceding Proposition*, and the 2<sup>d</sup> *Corollary* of the 13. *Infinity* is but a *Mode* in natural or created Things, but in the supreme or *absolute Infinite*, it implies in its positive *Idea* the utmost *Reality*, to wit, necessary Existence; and the same must of Necessity be true, in all those Attributes of the divine Nature, which *analogically* deriv'd down to the Creatures, are in them but *Modes*, in him

him they are the utmost *Realities*; because they are all complicated and affected with, or (so to speak) multiplied into *absolute Infinitude*, which *realises* those *Modes* of Creatures, and *transubstantiates* them into positive and *real* Qualities. So true is the *metaphysical Axiom*, *quicquid in Deo, ipse Deus est*. I do not here contend that there may not be *accessory Ideas* in the divine Intellect, or that there may not *arbitrarily* arise in the divine Intellect *Images* of Beings, whose Existence is Fact, is not necessary. Since the *Idea* of a Creature cannot be necessary to him, or else they would necessarily be. But even those *accessory* or *arbitrary Ideas*, in the divine Imagination, by being there, become of a quite different Nature from the like in Creatures, for by being there, they (by Virtue of his *absolute Infinitude*) receive a Being infinitely superior to the like *Ideas* in created Beings, not necessarily, but with infinite Freedom and Liberty. And those Affections and Properties in Creatures, which in them are but *Modes*, when *analogically* carried up, to the like or similar Affections or Attributes in the divine Nature, are in him the utmost *Realities*, as being complicated with *absolute Infinitude*, and thereby transform'd or exalted into real Quantities or actual Subsistences,

Corol-



## Corollary II.

HENCE *Power, Subsistence, Duration, Knowledge, Wisdom, Goodness, Beauty, &c.* which in created intelligent Beings are the Images of *Omnipotence, necessary Existence, Eternity, Omniscience, the divine Sophia, Benignity, infinite Perfection, &c.* in the divine Nature : And are but *Modes* of Being, and not essential Affections in these, are in him infinite *Realities*, and living active Principles. And he that wou'd reason *analogically*, from the Nature of these in created intelligent Beings, to the Nature of those in the divine Essence, without having the utmost Regard to the *absolute Infinitude*, which in a Manner quite changes the Nature of the former, and exalts them into a different *Category*, wou'd be miserably mistaken. For Instance, he that, because the Duration of natural Things is successive, wou'd conclude so of the divine *Eternity* : He that wou'd reason because human Power cannot give Being and Substance to that which had none, the divine *Omnipotence* cou'd not : He that wou'd infer, that because the Knowledge of rational created Beings is *progressive*, the divine *Omniscience* were also progressive, must be egregiously mistaken : Because in this *Analogy*  
he

he does not take in the *absolute Infinitude* which elevates and exalts the Duration, Power and Knowledge of created Things, into a Degree of Reality of which these are but the Images or Pictures. He that from the Picture of a Man wou'd *analogically* reason about human Nature, from the blending and Position of some Colours on Canvass, wou'd reason to Life, and Knowledge: Or from the reflected Image of the Sun in Water, wou'd conclude of Light, and Heat, cou'd not err more grossly.

*Scholium.*

IN intelligent compounded Beings: The *Powers* belonging to the Body are not only finite, but very low in the Order of Finities. The *Eye* perceives not distinctly a very large nor small Object. Too strong a Noise stuns the *Ear*, and one too weak does not act upon the *Organ*: Neither of them produces a distinct Hearing; and it is so in all the other Senses. Those *Organs* are so contriv'd, as to perceive best the ordinary Effects of common Life, the Objects that Necessity of Subsistence do most readily present to us. These they are fitted for, and little else; they have a wonderful Facility in manifesting these distinctly to our Minds, and have a just Proportion to the Objects  
about

about us, but seem not contriv'd nor design'd for Curiosity, or conveying much more Knowledge to us, than what the Conveniencies of Life require: Else infinite Wisdom and Power might have easily formed them so, as not to need those Helps and Assistances, which we are obliged to employ when we aim at any more particular Knowledge of the intimate Natures of the Things about us, than Conveniency makes necessary or commodious. The *Faculties* belonging to the rational Soul are likewise finite, but of a higher Rank of Finites than those Powers belonging to the Body. The *Imagination* can paint a larger *Idea*, than the *Eyes* can see, and the *Memory* lodge a greater Store of Images, than all the Senses can present at one Time: And the Understanding can combine and disjoin these, and compare them many different Ways: But still these Faculties are but finite in their *Capacity*. We can form no distinct Ideas of *Millions* of *Millions*, of a *multiangular Figure*, or any *relative Infinite*, small or great: The Imagination seems not able to contain these; there is no room on it, for so large Pictures. The *Memory* is the *Repository* of the Images that have been framed on *Phantasy*, and can go no further than it reaches; nay seldom or never contains more than a small Part of those. The *Understanding* can work  
no

no further than these two afford Materials; its Works being to *collate, combine, and garble*, as it were, these Images and *Ideas* the *Imagination* and *Memory* present to it. All these are limited, as the Senses are, tho' not quite so straightly, because the *Understanding* may variously combine those *Ideas* they have convey'd to the *Imagination* and *Memory*, and so increase their Number: And the *Will* having no subject Matter to proceed upon, but as it is prepared by these prior Faculties, can go no further than those allow it. The *Understanding* may diversify these as far as their *Combinations* will reach, and the *Will* may pick and choose among these, but since it cannot *create* Objects for it self to work on, it must be limited to the Images and *Ideas* on the *Imagination* and in the *Memory*. The *Will* I mean, as it is the Faculty belonging to the *rational Soul*. All these Faculties seem to have been originally design'd for Nothing but this *material World*, and the System of Things about us. They help us to no Notion or Conception of any Sort of Beings distinct from *Matter*, but in so far as *Analogy* will bear us out, and even as to the *material World*, they seem fitted for little else, besides the grosser, more general and more necessary Knowledge of Things that are required for due Conveniency, Sub-

sistence;

sistence, and shew but some of the grosser *Out-lines* of the real Natures of Things. Every Thing that might violently entertain our Curiosity, or flatter our Vanity, as to the Knowledge of the Nature of Things, seems to have been industriously conceal'd from us, and no Faculties to have been indulg'd us for these Purposes, least they shou'd have withdrawn us too strongly from Things of greater Moment to the End of our Being: Else infinite Wisdom and Power had contriv'd them after a more perfect Manner, with a larger *Capacity*, and a stronger *Energy*. As to the Faculties of the *supreme Spirit*, (which is a third Part of intelligent compounded Beings) they most certainly are infinite in their *Capacity* and *Energy*. I mean they may be enlarged and increased without Bounds or Limits, which by *Def.* 4. is to be relatively infinite. Not only the *Acts* of these Faculties may be multiplied perpetually, but the *Capacity* and *Energy* of these may be dilated and intended without Bounds or Limits. The *Perception*, the *Desires*, the *Will*, (the Faculties belonging to the *supreme Spirit*) are unlimited and boundless, fitted and design'd for *infinite Objects*. These indeed are the first, *principal*, and *original* Faculties, belonging to all compounded intelligent Beings: By which they are made capable of communicating



with the *supreme Infinite*. And next in order of Nature to the *supreme Spirit*, is the *rational Soul*, whereby they are enabled to communicate with the *material World*: And to the Faculties of this *secondary Part* of the Composition, the Senses of the Body are the *Conduits*, and Conveyances, which make up the third and last Part of compounded intelligent Beings: In the due *Subordination*, the perfect *Harmony*, and perpetual *Concord* of these Three, with each other, the *Perfection* of these Beings does consist: In their *Discord*, *Confusion*, and *Rebellion* one against the other, their *Degeneracy*, *Corruption*, and *Fall*. It can be no Difficulty to those who are acquainted with the *Analogy* of Things, to conceive how these several Parts of compounded intelligent Beings are contain'd without Confusion or Contrariety in each other; in their primitive and uncorrupted State: When they consider, that in Water is contain'd Air, in that *Æther*, in that Light, and perhaps in this last, a more subtle and refin'd *Spirit*; and all these in perfect *Harmony*, and *Concord*. But to consider these a little more particularly. The *Perception*, as it belongs to the *supreme Spirit*, must of Necessity be a *passive Principle*, because it cannot create its Objects, but receive those presented to it: That it is infinite, is plain, because its adequate Object is infinite, and the

*supreme*

*Supreme Infinite* : It being bestow'd on intelligent Beings, in order to communicate with the *absolute Infinite*. The Necessity of this third Part of the Composition of the mention'd Rank of intelligent Beings shall be afterwards demonstrated; I proceed to consider in a few Words the Nature of some others of these Faculties, belonging to the *supreme Spirit*.

*Proposition XV.*

THE *Desire* is infinite in its *Capacity*, the most *cardinal*, most *quick*, and *sensible*, and most *active* Faculty of the *Mind* or spiritual Part of compounded intelligent Beings, and the *Will*, and the *Affections* are but *Modifications* of it.

*Demonstrat.* To be convinced of the Truth of this *Proposition*, we need only reflect on the Source of all the Happiness or Misery of intelligent Beings, and we shall find it arises from the Enjoyment or Disappointment of their *Desires*. There is in all intelligent Beings a restless *Appetite* or *Desire* of Happiness : From the Moment of their Being, through all the Ages of Eternity, all their *Labour*, and *Travel* is for this Purpose : Nor are they devoid of it, either immediately in the *End*, or mediately in the *Means*, for one Instant of Time, in all their

endless *Duration*. Now this is the necessary Effect of the Faculty of *Desire*, no Object less than *Infinite* can satisfy it. For let it be supposed to have come to the Possession of any Object less than *Infinite*, its plain, it can *desire* yet a greater, and a greater, without Bounds or Limits; that is, it can desire an *infinitely great Object*, that is, the *Desire* it self is infinite in its *Capacity*; its *Acts* are *instantaneous*, and its Enjoyments or Disappointments for a Time swallow up the *Acts* of all the other Faculties: And therefore it is the most *quick* and *sensible*: It sets all the Powers of the whole Composition on Action, to obtain its *Ends*, and therefore it is the most *active*: And upon all these Accounts, the most *cardinal* Faculty of the *Mind*: Choosing or refusing, that is *willing*, is but the Desire apply'd to a particular Object; the *Affections* are but the *Complexions* of the *Desire*, as apply'd to this particular Object: And therefore are both but *Modifications* of the *Desires*, wherefore, &c. q. e. d.

*Scholium.*

To apprehend how infinitely capacious, active, and sensible the Desire is, we need only imagine our selves, separated from the Objects of Sense, and the present *Amusement*  
of

of *Life*, with all the Faculties of the Soul awake: And we shall then be able to conjecture how *strong, active, restless, and unsatiable* our Desires wou'd be. So as to swallow up and extinguish all the other *Acts* of the Faculties of the whole Compound. Those only can most sensibly feel the Force of this Reasoning, who have in some Measure, and for some Time been in this State.

### Corollary I.

SINCE the Desire is *infinite* in its Capacity, and may be dilated beyond any finite Object, how great soever, it is evident it cannot be over-fill'd, or super-abundantly (so to speak) satisfied with any Object less than *infinite*: Since also, the greatest relative Infinite, cannot be assignable by *Corollary 1. Prop. 1.* Therefore the *Desire* cannot be adèqually and over-fill'd with any less Object than the *absolute Infinite*; it being capable of being enlarged beyond the Dimensions of any *relative* or *creaturely Infinite* assignable: And since by *Corollary 6. Prop. 12.* no *Creature* can be *absolutely Infinite*, therefore the Desire can be perfectly and adequately fill'd and super-abundantly satisfied, by nothing less than the *supreme and increated Infinite*,

## Corollary II.

SINCE the *Desire* is *infinite*, when fill'd and satisfied with its proper and *adequate Objects*, it must be infinitely happy; for since Nothing, by the *preceeding Corollary*, but the *absolute* and *increated Infinite*, can adequately fill, and super-abundantly satisfy it, The *absolute* and *increated Infinite* must be its proper Object, and the *Desire infinite* in it self, fill'd and perfectly satisfy'd with its proper Object, the *supreme* and *increated Infinite*, must of Necessity be infinitely happy. Happiness arising from the *Congruity* of the Object with the Faculty, and this *supreme* and *increated Infinite* being the proper, and indeed the only proper Object, (since the *supreme Infinite* is one) it must alone be the *congruous* Object, and so the Faculty being *infinite*, the Object *infinite*, and they infinitely *congruous* to one another. The *Desire*, in the Possession or Enjoyment of this Object, must of Necessity be infinitely happy.

## Proposition XVI.

IN regard of *intelligent Beings*, every Thing that is in the *Universitas rerum*, may be fully comprehended under these three general



neral Heads, to wit, the *Faculty* or *Desire*, the *Object* of this *Faculty* or *Desire*, and the *Sensation* arising from the *Congruity* or *Incongruity* between these two.

*Demonstrat.* All the distinct Subsistences that actually exist, are comprehended under these two general Heads, the *supreme* and *absolute Infinite*, and the *Creature*, and these can be only Objects of the *Faculty* or *Desire*. The *Faculty* or *Desire* being infinite, by the *preceeding Proposition*, may contain or receive both these, and the *Sensation* arising from their *Congruity* or *Incongruity* to the Faculty, must together with these two general Heads mention'd, comprehend every Thing in the *Universitas rerum* in regard to any particular intelligent Being. For nothing can be imagin'd in the whole Extent of Being, *real* or *intellectual*, that may not be reduced to *Faculty*, *Object*, or *Sensation* arising from these. Therefore, &c. q. e. d.

*Proposition XVII.*

AN *intelligent Being*, compounded of a Body, Soul, and Spirit, with proper Relations, and in Subordination to each other, is a real *Epitome*, *Image*, or *Representation* of the *Universitas rerum omnium*.

*Demonstrat.* This is so evident from the *Analogy* of Things; that there can be no

Difficulty in it, to those who will consider, that the *supreme Infinite* cou'd have nothing more perfect than himself and his other Works, and his own perfect Administration and Government of these; to form this compounded Being upon. He must be consistent with himself, and his *compounded Works*, in their component Parts, must resemble in the *lesser* Compositions, the *similar* Parts of the *Greater*. This *intelligent compounded Being*, being to have a *material* Part, what can it have more worthy of the Work of infinite Perfection, than his *greater System* of material Beings to resemble? His Soul or rational Part can resemble nothing more worthy of *Him*, than the other higher Orders of created Spirits: And his *supreme Spirit* or Mind, will bear its best Resemblance, when it represents the *supreme Infinite*. His Administration and Government of the whole *System* of created Beings, can be represented by nothing so aptly, as the Relations and Subordination of these to each other, and to the rest of intelligent Beings, which these cou'd only be formed upon. In a Word, it is impossible that any *Idea* of a compounded intelligent Being, made up of several Parts, each *similar* to some greater Part of the *Universitas rerum*, already existent, cou'd enter to the divine Mind, which cou'd be more perfect and more worthy

thy of *Him*, than this, that each interior Part shou'd be an *Epitome* and Resemblance of his Works already made, and the highest Part of *Himself*, and that their Relations and Subordination shou'd resemble his Administration of the whole. I say it is impossible it shou'd be otherwise, to preserve his Consistency with himself, and to carry on this *Analogy*, through every individual Part of his Works, through the whole *Scale* of Beings, as it most certainly is carried. And this is not a meer *metaphorical* Picture, and Resemblance, but the real and physical Nature of compounded intelligent Beings. Wherefore, &c. q. e. d.

Corollary I.

FROM this Foundation, by a proper *Analogy*, with the due Limitations; all the *Relations* and *moral Duties*, of *intelligent compounded Beings*, to the *supreme Infinite*, to *similar intelligent Beings*, and to *themselves*, may be easily deduced; for Instance, do we resemble the *supreme Infinite*, in our *supreme Spirit*: Then as he is the first in *Order of Beings*, so is this Part of our Composition, the first and most principal, in our *Order of Parts*. All the rest must be subservient and subordinate in us, to this, as the rest of the several Ranks of Beings are to him: This

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we are chiefly and mainly to cultivate by fitting it up for him, for whom it was *originally* design'd and bestow'd upon us. Here, we must aspire after *Him*, and open our *Desires* for *Him*, by a *Love* worthy of *Him*, superior infinitely to all our other *Loves* and *Desires*: On this Part and its Faculties, our greatest, our chief, nay, and our only Labour is to be bestow'd; that it may be expanded, dilated, and elevated to its proper Rank in the *Order* of our Parts; that the due Subordination may be restored; which done, all the other Parts will perform their proper *Functions*, in *Harmony* and *Concord*. In regard to other *intelligent Beings*, we are to consider them as like *Images* of *Him* and his *Works*; and follow them with a *Benevolence* proper to such *Images*, to imitate his Conduct of *Love* and *Forbearance*, to all his Creatures. But this only by the by.

#### Corollary II.

HENCE, in a proper *Analogy*, the Nature of the *spiritual* and *material* Parts of compounded intelligent Beings and of the *Union* between these, as far as they may be known by mere human Reason, is to be deduced. Compounded intelligent Beings are *Epitomes* or *Images* of the *Universitas rerum*. In their

their Bodies they resemble the *material System* of Things, in their *spiritual Parts* they resemble the spiritual World, the Union of these two is a Resemblance of (or is maintain'd and preserv'd after) the Manner, the *supreme Being* governs the *material System* of Things: Who being intimately present with every individual *Atom* of Matter, yet more *eminently* acts from his superior *Throne of Glory*, having the whole *System* of Creatures in one View presented to him, in the *universal Space*, his special *Sensorium*. By this Principle, as a *Key*, the whole *Philosophy* of human Nature, of the animal, rational, and divine Life, of the Passions and Affections of the Soul, and even of the *Organism* of the Body, so far as it is just and genuine, and given to mere human Reason to know, is to be unlockt, and that not *metaphisically* but *phisically* and in reality. But who is sufficient for the detail of these Things?

### Corollary III.

HENCE, the *Immortality* of the spiritual Part, of intelligent compounded Beings, is evidently to be deduced. For since the *spiritual* Part, of intelligent compounded Beings, is an *Epitome* and *Image* of the *spiritual World*; and the *supreme Spirit*, (that Part



Part which God *originally breath'd* into Man) is an *Epitome* and *Representation* of the *absolute Infinite*; since by *Corollary 2. Prop. 13.* he necessarily exists, therefore the Soul, or *spiritual* Part (whereof this supreme Spirit is the *Fund* or *intimate Substance*) must exist for *ever*. Not necessarily, for that Conclusion wou'd drop the proper Limitation in the *Analogy*, arising from *absolute Infinity*: But, as being *Images* of him, who exists *necessarily*, and having their Being from him. Their *Immortality* is indeed an *active, living Principle*, not of necessary, (but *deriv'd*) Existence. They are *immortal*, as having represented on them all his communicable Perfections; of which, *perpetual* Existence is one, tho' *necessary* Existence be not; that being contrary to *Creation*, or deriving a Being from another. *Immortality* in Creatures is nothing but *perpetuity* of Existence, and if Existence at all be communicable to Creatures, *perpetual* Existence must be communicable: For Existence, being an *active, living Principle*, will of it self continue *Being* for ever, unless it be destroyed: Which is impossible, both from the *Immutability* of God, and the Nature of his own *Immortality*, of which this is an *Image*, or *Resemblance*. The *Immortality* of spiritual Creatures, is an *Emanation* from an *Image* of the divine communicable

ble *Immortality*. And must resemble every Thing in it, but *Necessity*, that being a Contradiction to its being *deriv'd*. But in every other Circumstance it perfectly resembles the *Immortality* of the superior Infinite, (as far as creaturely Properties can resemble divine Attributes) and so cannot possibly be destroyed: And this is the true Source of the *Immortality* of all Creatures. It is true, this *Demonstration* equally concludes the *Immortality* of all his other Works, since they are equally his *Images*, in a higher or lower Degree of Perfection: And without all doubt it must be so, *for the Gifts and calling of God are without Repentance*. And this is the genuine, and as the *Schools* call it, the *apodeictick Demonstration* of the *Immortality* of all the Works of God, under some Form or another; it is from their being his *Images* that their *Immortality* springs.

#### Corollary IV.

IN the *Analogy of Things*, the *Desire* being the *cardinal Faculty* of the Soul, and *infinite* in its *Capacity*, is as the *infinite Space*, to the *divine Plenitude*: Which *infinite Space*, nothing created can adequately fill, but the *divine Plenitude*. And in this View, the *infinite Capacity* of the *Desire* may be considered as a *boundless Void*, made  
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to receive some *fleeting*, limited Parts or *Systems* of Matter, acting by established Laws, and in comely Order, upon one another for a while, but can never be perfectly fill'd, or adequately replenish'd, but by the *supreme Infinite* : Who is present with, and replenishes every Point of the great and *universal Void* of Nature.

### Scholium I.

As in regard to intelligent Beings, the *Universitas rerum omnium*, is fully comprehended under these three general Heads; the Faculty or *Desire*, the *Object*, and the *Sensation* arising from the *Congruity* or *Incongruity* between these. So the same *Analogy* with proper Limitations is preserved in the material *System* of Things. For answerable to these, we have in the *material World*, *Gravitation*, which wonderfully *analogises* to the *Faculty* or *Desire* in the *spiritual World*, and this to that; both being the *active, cardinal, and energetick Principles* of either *Systems* respectively. Next we have a *Mass* of extended sensible Matter, if collectively considered, or *Systems* of *material Bodies*, diversly figured and situated in regard to one another, if separately considered : And in both these Views, they admirably represent the Subject or Object of the

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*Desire*, which is analogised by *Attraction* or *Gravitation*. And lastly, we have the *harmonious, comely, and decent* Motions, and Actions of one Body upon another, arising from the *Attraction* or *Gravitation's* being impress'd on the *Mass*, and each particular Body; and this too answers wonderfully to the *third* Principle in the *Universitas rerum*. And this *Analogy* descends even to particular Bodies, for in these we have the Form, the subject Matter, and the *Congruity* between these.

### Scholium II.

FROM the whole proceeding Chain of *intellectual Truths*, we may form to our selves some *faint, low, and imperfect Image* or Representation of the EVER-BLESSED TRINITY IN UNITY. For since by *Corollary* 1. *Prop.* 14. the *Modes* of natural or created Things, when *analogically* elevated to their similar Attributes in the *supreme Infinite*, in *Him*, are infinite *Realities*. Since, by the *same Proposition*, an intelligent Being is, as to its spiritual Part, an *Emanation* from, an *Image* and *Representation* of, the *supreme Infinite*: And since also, in regard to these intelligent compounded Beings, all that is in the *Universitas rerum omnium*, may be fully comprehended under these three general  
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Heads or Principles. The *Desire*, the *Object*, and the *Sensation* arising from the *Congruity* or *Incongruity* between the *Faculty* and the *Object*. Since lastly, this *Analogy* is preserv'd full and clear, through the *Spiritual* and *material* Worlds, and each particular Body, that is, through the whole *System* of Creatures : It is highly probable, so constant and universal an *Analogy* can arise from nothing, but from its *Patern* and *Architype* in the divine Nature : And without all peradventure, cou'd we fully and clearly carry it up to its *Source* ; we shou'd there find the *Origin* and *Source* of this HOLY TERNARY ; or of these three essential Relations, of the whole, to the whole in the completion of the GODHEAD. Let us therefore try, in some poor low Manner, to carry up this *Analogy*, as high as possible we can. The first Thing then that we are to consider in the divine Nature is, the *Desire*, now this being supposed, to belong to a *supremely infinite intelligent Being*, must be an infinitely *active*, *ardent*, *strong*, and *powerful Thought*. And that, not as created, or *relative Infinitude* expresses it, but as the absolute and *supreme Infinitude* elevates this *Desire*. Now then this supremely infinite *Desire*, this *active* and *ardent Thirst* after Happiness, or after a full, pleenary, and compleat, beautifying Object, we shall



I shall suppose to represent the FATHER, the *original* and first *Principle* in the divine Nature: Seeing then, there is no Consideration here of any Thing but of *God himself*, nor is it possible for any other Object, but *God himself* to satisfy, and adequately to fill this *supremely infinite Ardor, Thirst, and Desire of Happiness*. Therefore *He himself*, reflected in upon *Himself*, viewing and contemplating his *own* infinite Perfections: The *infinite ardent Desire*, fill'd and satisfy'd with his own essential Happiness: The *Brightness of his Glory*, and the *express Image*, (the essential *Idea*) of his *Substance*, reflected in upon *himself*: Or *God himself* reduplicatively contemplating *himself*, (he having nothing else possible, to be the Object of his *Love, Delight, or Desire*) represents to us the *begotten Deity*, the SON, the second divine *Principle* in the Order of the *Godhead*. Upon this *Reflection, Contemplation*, and Possession of *God himself*, in *himself*; there must of Necessity arise a *Joy, Happiness, Acquiescence, and Satisfaction* of *God himself* within *himself*, so much the more *perfect, full, extreme, and infinite*, as his *Desire, Thirst, and Ardor* after Happiness was *active and strong*. And this may shadow out to us the third and last in Order of these essential *Principles* in this *mysterious Ternary*, to wit, the HOLY GHOST.

PART II.

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And tho' these three Relations of the *Godhead* in it self, in regard to Creatures, and when deriv'd down *analogically* to natural Things, may appear but *Modifications* of a real Subsistence, yet in regard to the *divine Nature*, and considering his *supreme Infinitude*, they must be *essential* and infinitely *real* and *living Principles*; and in this *Image*, and View of the HOLY AND UNDIVIDED TRINITY, *low* and *poor* as it is; it is impossible the SON shou'd be without the FATHER, or the FATHER without the SON, or both without the HOLY GHOST. It is impossible, the SON shou'd not *necessarily* and *eternally* be begotten of the FATHER, or that the HOLY GHOST shou'd not *necessarily* and *eternally* proceed from both; *He* necessarily arising from the *Sensation* of the infinite Agreement and *Congruity* of the *Object* with the *Desire*. And tho' the *Idea*, *Image*, or *Representation*, that *God* makes of *himself* to *himself*, be the same *God* in *Essence*, since it is a most perfect, express, and substantial *Image*, or *Representation* of the whole divine Substance and Nature, and *necessarily* has in it the whole, that is in *God*, and with the same infinite Perfections, that is really in the *Godhead*, being only the *Divinity*, reflected in upon it *self*: Yet there is a Difference, between the *Idea*, *Image*, and *Representation*.

presentation, and the *original Divinity*. For the same Perfections which are in the *original* and *contemplating Divinity directly*, are but *indirectly*, and by Reflection in the *contemplated* and *begotten Deity*; and they differ by the *Relations* of *begetting*, *contemplating*, and *representing*; and being *begotten*, *contemplated*, and *represented*: And tho' these be only *mere Relations*, and *Modifications*, when transfer'd to natural and created Things, yet are they *infinite Realities* in the *Deity*. It is the same with the *Sensation* of *Love*, *Joy*, *Acquiescence*, and *Happiness*, that arises in the *divine Nature*, from *contemplating* and *possessing himself* and his own infinite Perfections within *himself*. And these *three Relations* of *contemplating*, being *contemplated*, and of *Acquiescence* arising from them: Which in a natural View, wou'd only be *Modifications*; yet in the *divine Nature*, are infinite *Realities* and *essential* and *living Principles*: And may serve as a faint and imperfect *Image* of THIS ADORABLE AND UNSEARCHABLE MYSTERY. Now tho' these Views and Representations of this INEFFABLE AND INCOMPREHENSIBLE MYSTERY arise *naturally* and *necessarily* from the *Analogy* of Things, sufficiently established in the *preceding Propositions*; and without all doubt, have some *real Truth* in them; since it is certain, these

*Relations* and *Modifications* of Subsistence are really in *intelligent Beings*, in the *material System* of Things, and in *particular Bodies*: And cou'd have no other *Rise* and *Source*, but from their *Patern* and *Architype* in the *divine Nature*. Yet when apply'd to the *supreme Infinite*, these *Relations* and *Distinctions* must be express'd in Words, that have a quite different Sense affix'd to them, by common Use, from what they shou'd here *import*: And since the *Attributes* and *Relations* of the *supreme Infinite*, must be *incomprehensible* to *finite Creatures*, especially to *Creatures*, conversant only about low, gross, and material *Images*; all I wou'd be understood to conclude from this *Analogy*, thus rais'd to the *supreme Infinite* is, that *Reason* may form an *analogical* imperfect *Image* (and that's all it can do) of this INCONCEIVABLE MYSTERY, which may in some Measure help those (who have not attain'd to a more perfect Guide or higher Lights) to believe the positive *Relations* of his own Nature, by God himself, tho' they be not able perfectly to comprehend or express them. And if this poor Representation of so PROFOUND A MYSTERY, so *certainly* and FULLY reveal'd in *holy Writ*, can, by the *divine Blessing*, any wise contribute to *settle* and *quiet* sober and honest Minds, I shall have my full

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Intention. Nothing less than *Omnipotence*, and *Omniscience*, being sufficient to deal with those who are otherwise disposed.

*Proposition XVIII.*

THERE must of Necessity be some *Principle* of *Action* in intelligent Beings, *analogous* to that of *Attraction* in the material *System*, and that is, the *Principle of Reunion* with the *supreme Infinite*, by him originally impress'd on their *supreme Spirits*.

*Demonstrat.* That there must be some Principle of *Action* impress'd on intelligent Beings, *analogous* to that of *Attraction* in the material *System*, is evident from the *Analogy of Things*, the Consistency of the Works of the *supreme Infinite* with themselves, the *Uniformity* conspicuous in all the Creatures, and that the Material are but *Images* and *Representations* according to their respective Natures of the *supreme Infinite*, as well as of the intelligent *System* of Beings. There must therefore some *great Principle* of *Uniformity* run thro' both *Systems*, that is, the whole Creation. Now that *Attraction*, or something *analogous* thereto, is the great Principle of *Activity* in the material *System*, has been sufficiently demonstrated in the former Part of this Treatise. There must of Necessity therefore be some *great Prin-*



*Principle analogous* to this, in the *System* of intelligent Beings. And that this can be nothing, but that great *Principle of Reunion*, with the Author of their Being, originally impress'd on every intelligent Creature, is evident from hence, to wit, that the *supreme infinite Being*, infinitely powerful and perfect, must necessarily subject, draw, and unite all intelligent Beings to himself, to make them as happy, as their respective Natures can admit. That himself is the *sole Object* of their Happiness cannot be doubted; there may be collateral and accessory Objects of their Happiness, but even these must flow from him: But that he is the supreme Object, and the single one that can adequately satisfy them, has been shewn before: That therefore, to bring them to this Happiness, he must impress upon, or derive to them a *Principle of Reunion* with himself, is plain, because this is the single Mean, to bring them to this End; for the known Law of Nature obtains even here, and every where, to wit, that *Action and Reaction is mutual*, so that if the *supreme Infinite* must draw them to him in Order to make them happy, they must have a Principle of being drawn towards him. That it must be a *radical* and *essential* Principle, is evident also, because this Happiness is the very End of their Creation, it being impossible in-  
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finite Perfection shou'd make intelligent Beings, for any less or any other *End*. Wherefore the *supreme Infinite* cou'd not make intelligent Creatures, without *implanting* in their Natures, a *most ardent Desire*, an *essential* Principle, interwoven in the Substance of the spiritual Natures, of being re-united with himself, in order to make them as happy as their relative Natures will admit of. Besides, intelligent Beings (as to their spiritual Part) are Images of the *supreme Infinite* by *Prop. 15*. In him there is an *infinite Desire*, and *Ardor* of possessing and enjoying himself, and his own infinite Perfections, in order to render him happy; he himself is the *sole Object* of his own, and of the Felicity of all his Creatures. There must therefore be an Image of this his infinite Desire after Happiness in all his intelligent Creatures; and this Image can be nothing but this *Principle of Reunion*, since nothing but this can unite them with him, to make them happy. Lastly, an intelligent Being, coming out of the Hands of *infinite Perfection*, with an Aversion, or even Indifferency, to be re-united with its *Author*, the Source of its utmost Felicity, is such a Shock, and Deformity in the beautiful *Analogy of Things*, such a Breach and Gap in the *harmonious Uniformity*, observable in all the Works of the Almighty, and that

too, in the noblest and highest Part of his Works; as is not consistent with finite Wisdom and Perfection, much less with the *supremely infinite* Wisdom of the ALL-PERFECT. Wherefore, &c. q. e. d.

Corollary I.

HENCE we may discover the Source of *natural Conscience*, and of all those Motions and Convulsions, that are raised in the Breasts of compounded intelligent Beings, upon the *Commission* and *Omission* of certain Actions: Of that *Comfort*, *Joy*, and *Support* in some; and of that *Dejection*, *Dread*, and *Terror* on the Minds of others; where no natural Causes can be assign'd. Hence the *noble* and *sublime Discoveries* of the *anti-ent Heathen Philosophers*, in the Principles of moral Virtues, without the Assistance of Revelation. Hence it is, that *Scelerats* can by no Arts, nor any Amusements how violent soever, stifle the Cries of a wounded Conscience; and hence also, it is, that honest and upright Minds are sometimes swallow'd up by a Tranquility and Peace that surpasses natural Understanding. That this *Principle of Reunion* is defaced, buried, and in some Measure as it were obliterated by contrary *Attractions*, by *Sensuality*, and the violent Amusements of Licentiousness in compounded

pounded intelligent Beings, is no more an Argument against its *essentially* belonging to intelligent Beings, than the *Ideotism* of some is an Argument against the Principle of Reason in human Nature. But,

Corollary II.

FROM this *Principle's* being so *radically* implanted in all the Individuals of intelligent Beings, and from the so very few Instances and Remains of it, conspicuous in the human Race; we may gather the infinite and universal Degeneracy and Corruption of this Set of intelligent Beings, from their *primitive* and *original* Institution. This Principle was most certainly implanted in their Creation, in the very *Fund* and Substance of their Natures, and yet there remains but few Footsteps, and Instances of its *Being* or *Effects*. There are indeed (as in the greater World) some *legible Characters*, *strong Out-lines*, and *prominent Lineaments* of its original Beauty, some *magnificent Ruins*, which shew what it had been, enough to demonstrate the *original Impression*, Beauty, and real Being, of such a *Principle*, in all the Individuals of this Race, as is evident from the *Pangs* and *Tor-  
tures* of *natural Conscience*, when it is counter-acted. But the little Effect it has, from  
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what it was design'd to have, is an evident Demonstration of the deep and universal Corruption of this *Set* of intelligent Beings.

Corollary III.

HENCE we may farther discover the Force, Truth, and Universality of this wonderful *Analogy of Things*; whatever we discover of the Works of Nature, is from this Source, and whenever we get the least *glympse* into the Manner of the divine Operating, we discover fresh Instances of this *Analogy*. This *Principle of Reunion* in intelligent Beings, wonderfully *analogises* with that of *Attraction* in the material World: As to the *supreme Infinite*, it may be very properly called his *Attraction* of them, and as to them, their *central Tendency* or *Gravitation* (so to speak) toward him; and this *Principle of Reunion*, if attended to, duly cultivated, and expanded, wou'd as certainly bring about the temporal and eternal Happinels of all intelligent Beings in the *spiritual World*; as that of *Attraction* brings about the comely and *harmonious* Motions of the *great Bodies* of the *material World*. This *Principle of Reunion* is the *original* Source and *Spring* of the *Desire*, afore-demonstrated to be the *cardinal* Faculty of the *spiritual Part* of intelligent Beings.



Beings. The *Principle of Reunion* is the *Root* and Foundation of the *Desire*, in regard to its first Cause and original Impression by the *supreme Infinite*. This *Principle of Reunion*, as it is considered, as an infinitely *active, quick, and sensible* Faculty, in the spiritual Part of compounded intelligent Beings, is the *Desire of Happiness*; as it is considered as a *theological* Virtue, it is *Charity*; and as it is considered as a *Rule* of Action, it is *natural Conscience*. But the *Principle of Reunion*, in its whole Extent, as it was originally impress'd upon, and quite interwoven with the intimate *Fund* and Substance of the spiritual Part of compounded intelligent Beings, is the *Source, Origin, and Root* of all these.

#### Corollary IV.

HENCE, the true and genuine Nature of *moral Good* and *Evil*, and of all the moral Virtues, and *social* Duties of Life, as from their genuine Fountain and Source, is to be derived. Whatever retards, or opposes this *Reunion* in intelligent Beings, is to them *moral Evil*, whatever promotes or advances this *Reunion*, is to them *moral Good*. Besides, this *Principle of Reunion* duly cultivated, regularly unfolded, and carefully attended to, must necessarily produce and perfect

fect in the Soul *universal Charity* ; that is, the Love of the supreme Being, and of all his Images in a due Subordination ; and thereby instruct, and beget in the Soul, all the *moral Virtues* and *social Duties* of Life. All these being *virtually* and necessarily included in *Charity*. But to explain this Matter (which is of the utmost Consequence towards a right Apprehension of *solid Religion*) a little farther, I say

*Proposition XIX.*

*Charity*, or the Love of the *supreme Being*, and of all his *Images* in a proper Subordination, according to their *Rank* in the *Scale* of Subsistences, is the necessary Effect of this *Principle of Reunion*, when fully expanded and set at freedom.

*Demonstrat.* By *Corollary 4. Prop. 17.* nothing but the *supreme* and *absolute Infinite* can adequately fill and superabundantly satisfy the *infinite Desires* of intelligent Beings ; the *Desire* in intelligent Being is their *Love*, for no intelligent Being can desire any Thing in Order to make it happy, but what it *loves* ; or can *love* any Thing, but what it *desires* to enjoy, (*Indifference* being the middle State, and *Aversion* the equal Opposite, both to *Love* and *Desire*.) So that an infinite *Desire* of the *supreme*  
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*and absolute Infinite*, is an *ardent Love* of that *Being*. But the *Principle of Reunion*, expanded, set at freedom, and arrived at its *ultimate End* and *Center*, is intirely the same with the *infinite Desire* in intelligent Creatures, possess'd of its sole and proper Object, the *supreme and absolute Infinite*. Consequently the *Principle of Reunion*, expanded, and set at freedom, in order to arrive at this *ultimate End* and *Center*, must necessarily beget in the Spirits of intelligent Creatures, this *infinite Desire* of the sole and proper Object; that is, an *infinite Love* of the *supreme Being*: And by necessary Consequence, a *Love* of all his *Images* in Proportion to their *Resemblance* of him; that is, the *Principle of Reunion* when expanded, and set at freedom, must necessarily beget in the Spirits of intelligent Beings, a *Love* of the *supreme Being*, and of all his *Images* in a proper Subordination, according to their *Rank* in the *Scale* of Subsistences, that is *Charity*.  
q. e. d.

Corollary I.

HENCE, *Charity*, or the *Love* of the *supreme Being*, and of all his *Images* in a proper Subordination, in its true and genuine Nature, is not founded on *Interest*, or  
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the *Views* of Rewards and Punishments; but altogether on the *abstracted Perfections* of its Object, the *supreme Infinite*. *Charity* in its Origin, and as it ought to be, according to the true *Analogy* of Things, is a *physical* and necessary Consequence of the *Principle of Reunion*: It flows naturally from an *implanted Faculty*, and has for its Object the *supreme Infinite*, in his own *independent* and *essential* Nature, as he is *absolutely* good and perfect, without any *collateral Views* or *Regards*. *Charity* is in a higher Degree, and in a more noble Creature (one, to wit, endowed with freedom and liberty of acting) what *Motion*, proceeding from the Principle of *Gravitation*, is in *Brute-matter*, or what the *Tendency* of the *Planets* is towards the *Sun*, viz. a natural Consequence of an implanted Principle. This is so clear and so certain in true *Philosophy*, that it is Matter of Astonishment to me, how it came once to be doubted, much less disputed. *Henry Moor's* Argument on this Head is as cogent and just, as any Demonstration in *Euclid* or *Apollonius*. "As the Object of the Intellect (says he) is that which is simply true, and is assented to as such, and not as true to this particular Intellect which contemplates it, so there is an Object that is simply Good and Lovely, and to be lov-

“ *ed as such, without regard to the Party*  
“ *that thus loves it.* And in another Place,  
he uses the familiar Illustration of *Ginger-*  
*Bread* and *Mathematicks*, wherein he grants  
the former may be a Spur to the latter,  
’till Age and good Sense, with the Know-  
ledge of the intrinsic Beauty and Worth of  
the latter, makes the *Student* in love with  
the *Study* it self, without any Consideration  
of the *childish Bait*. As in natural Love,  
Persons become often enamoured of *out-*  
*ward Beauty*, without any particular  
Knowledge of its Possessor, or its *attain-*  
*ableness* by them; so without all peradven-  
ture, *infinite Perfection*, for its own intrin-  
sic *Pulchritude*, must be the proper Object  
of *divine Charity*, without any particular  
regard to the Party loving it. Not that  
*Interest*, or a *View to Rewards* and *Pu-*  
*nishments*, is not often the only Motive of  
*divine Love*: And always is a very proper  
and *laudable* one, when *infinite Perfection*  
is the Object. And in reality, it is as high  
as most of the *lapsed Race of Adam*, in their  
degenerate State, can rise to. But as Things  
are in their *original Natures*, were in their  
*Integral*, and must be in their *reintegrated*  
State. *Infinite Beauty*, or *Perfection*, with-  
out any regard to *Self-Interest*, or any View  
to *Rewards* or *Punishments*, is, was, and  
must



must be, the proper and sole Object of pure and perfect *divine Love* or *Charity*.

Corollary II.

HENCE the *Service, Worship, and Homage*, we owe to the *supreme Being*, is founded intirely upon his own *original Excellencies*, and Perfections, and not on his *Rewards* and *Punishments*; there neither ever was, nor ever cou'd be any room for *Contracts*, or *Pactions*, between the *supreme Being*, and his intelligent Creatures, in the original Constitution of Things. He made all Things by the Word of his Power, and for his Pleasure they are and were created. *Infinite Perfection* is to be lov'd, admir'd, ador'd, and serv'd, for being infinitely perfect, *antecedently*, and without any regard to Creatures: And when Creatures are brought into Existence, this *primary Reason of Love*, and *Adoration* subsists, and in order of Nature and Dignity is *prior* and preferable to all other Reasons. *Creation* adds nothing essential to *infinite Perfection*, but a *Circumstance* only, which too intirely vanishes when brought into Comparison with the original Beauty of the *absolute Infinite*. All his Creatures, that act naturally, correspond, and are faithful, to the greatest Exactness, to his *original Impressions*, and his appoint-

ted and design'd Ends on them. The *Celestial Orbs*, the *Sun*, *Planets*, *Comets*, and *fix'd Stars*, those huge unweildy Masses of Matter, revolve in regular *Periods* and constant Order, by their impress'd Power of *Gravitation* and *primitive Frame*. The *vegetable Tribes* are faithful to his appointed Seasons. The *Brute Animals*, sacredly obey their *inbred Instincts*. Only his compounded intelligent Creatures, if they at all obey, do it for their own, not his sake, that is, they themselves are their own *ultimate End*, contrary to what was *Demonstrated*, *Corollary 4. Prop. 17*. It is true in the Nature of Things, it's absolutely impossible, to separate the *Duty* and *Felicity* of intelligent Creatures, their *Happiness* consisting in their being in the *Order* of their Nature, that is, of GOD it's *Author*, and their only *Duty* being faithfully to obey his *Order*. Yet these two may be considered apart, and *mentally* abstracted the one from the other : And the last must be *Anterior* to, and in the *Order* of Things, must precede the former : So that when we invert this *Order*, and bring in our *Happiness*, as the sole *Motive*, or the preferable *Motive* to our *Duty*, we become guilty of the most *gross and blackest Idolatry* : For he who loves and worships the *supreme Being*, only because his one *Happiness* is thereby

PART II.

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brought about, makes himself and his own Happiness his *ultimate End*, and so places himself in the *Rank and Order* that belongs to GOD only, and himself becomes his own *Idol*. He only can be said truly to *Love* GOD, with a *Love* undivided, and worthy of him, who having *loved him*, as far as *Rewards and Punishments* will carry him, goes on still further to *love him*, loses all views of these *mercenary Motives*, and seeks for no Fuel to feed the *celestial Flame*, but the *unexhaustible Pulchritude*, and Perfections of the *beloved Object*.

*Proposition XX.*

*Charity*, or the pure and disinterested *Love* of GOD, and of all his *Images* in a proper Subordination, is the *end of the Law*; the *Accomplishment* of all the *Graces*, and the consummate Perfection of *Christianity*.

*Demonstrat.* On these Two, to wit, the *Love* of GOD and our Neighbour, hang both the *Law and the Prophets*, says the ADO-  
RABLE AUTHOR OF CHRISTIANITY and the *Apostle* says, the *end of the Law* is *Charity*, if the *supreme Being* is the *ultimate Object* of the *Felicity* of all intelligent Creatures, and *Charity* the mean to attain this end, as is evident from the *preceding Proposition*; then is *Charity* the consummate *Perfection of Christianity*. The whole

whole of *Christianity* is nothing but Rules for attaining this *Love*, or Measures whereby to remove the Impediments that hinder this *Principle of Reunion* (the source of *Charity*) from operating, or Means to destroy the contrary *Attractions* which disturb the natural Operation of this *Principle of Reunion*; which wou'd of it self, if not stilled, opposed, and counteracted, necessarily beget this *Divine Charity*, whereby the Soul wou'd instantly be united with it's *Center*, and *ultimate End*, the *supreme and absolute Infinite*. q. e. d.

*Scholium.*

To this Doctrine of *pure Love*, there are made but two Objections which have any Weight or Force in them. The First is, that the *Motives* for *Love and Obedience*, urged by *Moses* and the *Prophets*, *CHRIST* and his *Apostles*, are founded on *Rewards and Punishments*, and that therefore without *Blasphemy*, we are not to offer at, or pretend to, more high and sublime *Motives* or Principles, than the *Friend of GOD*, and the *SON OF GOD*, (the *Standard* in their several *Dispensations*, of *Purity and Perfection*) thought fit to press or propose. The plain and genuine Answer to this Objection is, that the *Author of our Being*, who loves all his *Creatures* better

*than they can love themselves, uses all Motives that are honest, laudable, and just, to gain them; he knows perfectly the Frame and original Complexion of all his Creatures, and that in their lapsed State they must ascend to Perfection by Steps and Degrees; and consequently that some are to be wrought upon by one Motive, others by another, and that generally the first Steps are mounted by the Force of the Terrors of the Lord, before the Love of GOD is shed in their Hearts. There are Babes in CHRIST, as well as grown and perfect Men, and their Food (or Motives to Charity) must be as their Years and Strength are: But our SAVIOUR tells us, we must love the LORD our GOD, with all our Heart, with all our Soul, with all our Strength, and with all our Mind, and if so, we shall have very little Love left behind for our selves. And his beloved Disciple tells us, that perfect Love casteth out Fear, and consequently Hope, that is, Rewards and Punishments, which are true, good, and salutary Motives, tho' not the best. The second Objection is from the impossibility of Loving or begetting Love without a Regard to Rewards or Punishments. But this Objection arises from Ignorance of the true Nature of this Affection of the Soul. Love is the Complexion of the Will or Desires, as was shown Prop. 15. it belongs to the*

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*uninlighten'd Faculty* of the Mind, the *Will*, and not to the enlighten'd Faculty, as the *Understanding* is, and so naturally, and of it self has no real Respect to *Rewards or Punishments*, which are Motives offer'd by the Understanding: We *love*, because we will *love* without *Reasoning*, or because the Object of our *Love* is amiable, and not because it will hurt or heal us. *Love* is blind, and belongs intirely to the *Will*, and not to the *Intellect*. But passing this, as perhaps too *metaphysical*, I answer, 2<sup>dly</sup>, as we may for one single *Instant*, and for one single *Act*, abstract from a Reward, forget it, or counteract it, (which no Body who knows the prescindent Faculties of the Soul, and that *Love* and Rewards are essentially united in their own Natures can deny) so we may thus *abstract* again, and again, and so in *Infinitum*, and thus beget a *Habit* for what may once be done, may for any impossibility in the Nature of the thing be done *for ever*: This is *Demonstration*. But I proceed,

*Proposition XXI.*

IN all intelligent Beings, there must be *Faculties* fitted for all the several *Ranks* of Objects, in the *Universitas rerum*; that is, since there are evidently three *Ranks* of

Objects, in the *Universitas rerum* ; to wit, the *material System* of Things, the *spiritual World* of created Spirits, and the *supreme and absolute Infinite*. Intelligent Beings must necessarily be fitted with *Faculties* suited to these three generical Ranks of Objects.

*Demonstrat.* This is evident from the several different *Mediums*, which all infer the same Conclusion. 1. Intelligent Beings are *Images* of the *supreme Infinite*, who alone perfectly comprehends and knows himself and all his Creatures, that is, all these three generical *Objects*, in the *Universitas rerum*. He has different Sensations and Perceptions (as far as *Diversity* can be consistent with his infinite *Simplicity*) arising in himself from all these three different Objects, and consequently *Faculties* fitted for them: Therefore intelligent Creatures, his *Images*, the *Representations* of all his communicable Perfections, must of necessity have *analogous* *Faculties*, fitted for all these three different Objects. 2. There are *Relations*, incumbent upon all intelligent Beings towards each other, and towards the *supreme Infinite*; such as *Love*, and *Benevolence*. Therefore intelligent Beings must be endow'd with *Faculties* fitted for receiving the Impressions, and to perceive the Effects of these *Relations*, else they wou'd be in vain: And these are two of the different *Ranks* of Objects ;

Objects ; and no Body questions intelligent Beings, being fitted with Faculties for the third ; to wit, the *material System* of Things. 3. As to the *spiritual World*, including the *supreme* and *absolute Infinite* as its *Head*. The *Principle of Reunion*, whose necessity in intelligent Beings, I have now demonstrated, makes it absolutely necessary, they shou'd be provided with *Faculties* fit for *Communication* and *Union* with the *supreme Infinite* ; else, they were not susceptible of that Happiness which was the *sole End* of their Being. 4. That which is the *apodeictick Demonstration* of the Truth of this *Proposition*, and at the same Time proves the Diversity of these several Faculties, to be as real as that of the Objects is, is the manner after which compounded intelligent Beings are provided with *Faculties* for the *material System* of Things. Material Things are presented to them only through their Senses ; they have a real and material *influx* on these, else they are not really perceiv'd ; and all real Knowledge of material Things is convey'd into the Understanding, through these Senses : Wherefore it is evident, compounded intelligent Beings are endowed with a Faculty of perceiving or receiving material Things, thro' their Senses, which is called *Perception*. Next they have a Faculty of painting these Perceptions or

their Images when the Objects are absent, and this is called Imagination, and *lastly*, a Faculty of *combining* and comparing the real Perceptions of these material Things or their *Images*, and this Faculty is called *Reason*. And all these *three* distinct Operations belong to the *rational Soul*, in order to fit it for Communication with the material World. Now by the *Analogy* of Things, such like and *similar* Faculties must of necessity belong to the *spiritual* part of compounded intelligent Beings, to fit them for a Communication with the two remaining *Ranks* of Objects; to wit, a *spiritual Perception* and *spiritual Senses*, *Imagination* and *Understanding*, for the *spiritual* World of intelligent Beings, and *divine Senses*, *Perception*, *Imagination* and *Understanding*, for communicating with the *supreme Infinite*. For this *Analogy* will perpetually hold good and true, from the *Simplicity* and *Unity* of the *divine Nature*; to wit, such as the Faculties are by which we *communicate* with the *material* World, such *analogically*, and with proper Limitations, are those by which we *communicate* with the World of Spirits, and the *supreme Infinite*. And as we see compounded intelligent Beings have bodily *Senses*, which solely belong to the *material* World, so in the *Analogy* of Things, they must have Faculties different

different and distinct in themselves peculiarly fitted appropriated to the other Objects in the *Universitas rerum*. Wherefore,  
Ec. q. e. d.

Corollary. I.

HENCE we may conceive the Reason why in *holy Writ*, the whole Man is distinguished into *Body, Soul* and *Spirit*; whence comes the Distinction of the *natural* and *spiritual*, or *inward Man*, between the *Law of the Members*, and the *Law of the Mind*. These Distinctions and Divisions, I say, are easily conceiv'd from the foregoing *Proposition*. For the Body and rational Soul belong to this *material System* of Things, and are fitted with Faculties for *communicating* with it, and is called the outward Man; and the following it's practical Dictates in Rebellion against, and in Opposition to the Dictates of the *Spirit*, the *inward Man*: The essential *Principle of Reunion*, the *Law of the Mind*, which is fitted only for *communicating* with the *supreme Infinite*: I say, the following the practical Dictates of the first, in Opposition to the *Dictates* of the latter, is called in the Language of the *Spirit*, the following the *Law of the Members*.

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## Corollary II.

As there are *three* different Principles, Orders, or *Predicaments* of Being ; to wit, GOD the *Creator of all Things*, created *Spirits*, and *material Bodies* : So there are *analogically*, in compounded intelligent Beings, *three* different and distinct Principles, adapted and appropriated, for communicating with, and enjoying these respective Objects: Whereof every one is endow'd with proper *Senses*, *Powers* and *Faculties*, different and distinct from each other ; that is, as the *material System* of Things, is the proper Object of the Senses, and rational Soul ; and as this Principle is fitted with outward *Senses*, *Perception*, *Imagination*, *Understanding*, and *Will* ; So the Principle, whereby they are fitted to communicate with the created spiritual World, is endowed with inward *Senses*, *Imagination*, *Understanding*, and *Will* ; and the *supreme Spirit*, or third Principle, whereby they are fitted to communicate with the supreme uncreated Infinite, is endowed with *inmost* (so to speak) *Senses*, *Imagination*, *Understanding* and *Will* ; and all these in their *primitive* and *original* Constitution, in Subordination, *Harmony*, and Agreement, without Contrariety or Confusion one with another.

another. This *Corollary* is as certain as the *Analogy* of Things is.

*Corollary* III.

HENCE we may deduce the true and genuine Nature, and Extent, of the *Degeneracy*, *Corruption* and *Fall* of the human Race of intelligent Creatures, which consists in the *Confusion*, *Discord*, *Rebellion*, and *Contrariety* of these different and distinct Principles one with, and against another; in throwing off that *due Subordination*, *Subjection*, and proper *Rank* and Order, that was originally established among these Faculties; according to the Dignity of these different Objects; the Order of Nature, and the *Analogy* of Things, and following the practical *Dictates*, and Conclusions deduced from these rebellious Faculties. That is, when in this *anarchical* and rebellious State of human Nature, the Faculties belonging to the material World presume to judge of, and determine the Nature of the Subjects belonging to the *supreme* Spirit; take the Government and Administration of the whole Man, which properly belongs, in the Order of Nature, to this third Principle; lead the other Principles as *Slaves* and *Captives*, and force them to comply with the practical *Dictates* they prescribe,  
and

and deduce in their usurp'd Superiority; and under this compliance, begetting and producing physical and durable Effects, the whole Order of Nature, and the material *System* of Things, so far as these *physical* and durable Effects reach, becomes distorted, inverted, and corrupted.

*Proposition XXII.*

THE *rational Soul* is not that Faculty in compounded intelligent Beings, which in the Order of Nature, and the *Analogy* of Things, is appropriated for the spiritual World (including the supreme Infinite as it's Head.)

*Demonstrat.* Tho' this be a necessary *Corollary* from the preceding *Proposition*, yet since it is of great Consequence to true divine Knowledge, to have it's Truth established beyond all possibility of *Cavil*; I shall here suggest some other *Mediums*, or set those already suggested in another Limit, from whence the same *Conclusion* may be deduced. And 1. This is evident from the Nature of this *Faculty*, and the manner of its operating. *Reasoning* is the comparing, or the considering the *congruity*, or *incongruity* of the Perceptions, suggested by the Senses, or of the *Ideas* lodg'd in the Memory, or painted on the Imagination to one another; and *Reason* is the Faculty whereby

whereby this is perform'd. Now the *Senses* send in only the *Influxes* of material Things, and the *Imagination* and *Memory* present only their Pictures or *Images*, when the Objects themselves are absent; and here is all the *rational Soul* can do. But nothing of these belong to the *supreme* and *increated Infinite*, nor the *spiritual World*. 2. It is acknowledged by all, and every one's Experience demonstrates it to him, that the *rational Soul* is fitted for *communicating* with the *material World*. Now since Body and Spirit are precise and proper Opposites, it wou'd be as *dissonant* and *incongruous* in the *Analogy* of Things, that the same Faculty shou'd be the Principle of Communication with these two Objects so widely distant, as that the *Eye* shou'd both *bear* and *see*. It is true, it may be said, that the *rational Soul* might have been originally endowed with such *Energy* and Capacity, as to be fitted for *communicating* with *both Worlds*; but this is meerly *gratis Dictum*, and perfectly contrary to the *Analogy* of Things; it being impossible to bring an Instance of Nature, where Things so widely distant, and precisely opposite, are receiv'd by *one and the same* Faculty. We see in the *Body*, or lowest part of the human Composition that it is instructed with *Organs* fitted for all the possible Ways, material

terial Things can act upon it. And they are not near so widely distant and different in the manner of operating, as Bodies and Spirits in their Natures are. 3. There is a Two-fold Knowledge of material Things, one *Real*, when the Thing it self, and the *real* Action and Impression thereof on our Senses is perceiv'd: The other *Ideal*, when the *Image* or *Idea* of a thing absent in it self, is represented to, and considered on the *Imagination*: For Instance, the *Heat*, *Light*, and *chearful Influences* of the *Sun* shining on us, are widely different from the view and Consideration of its *Image*, or *Idea* on our Fancy. In the *Analogy* of Things, and according to the constant Order of *Nature*, such must our Knowledge of *spiritual* Objects be; to wit, the one *real*, when the Objects make a real Impression upon the *appropriated* Faculty; the other *Ideal*, when we frame a Notion of its absent Substance and Qualities. Now it is very plain, the *rational Soul* is not fitted for this first kind of Knowledge or Perception of *spiritual* Objects, since a great many *philosophical* and *learned Men*, who have exercised this Faculty in its greatest Strength and *Vigor*, have deny'd the Existence of such Objects. 4. Lastly, That most certain and self evident *metaphysical Axiom*; to wit, *that nothing can be in the Understanding, that*  
was



was not first in the Senses, is a certain Demonstration, that the *rational Soul* is not the Faculty in intelligent Beings, appropriated to the *spiritual* World; for every Body allows, that *spiritual* Beings, as such, can never be convey'd, through the bodily Senses, to the Understanding. And therefore we must either be intirely depriv'd of Faculties, for communicating with *spiritual* Beings (that is, we must be depriv'd of the only means of our *supreme Felicity*, and for attaining the End, for which alone we were created; to wit, communicating with the *supreme and absolute infinite Spirit*.) Or else, we must be endow'd with Faculties distinct from the *rational Soul* for that purpose: Upon all which Accounts it is evident beyond the most remote possibility of doubt, that the *rational Soul* is not that Faculty in compounded intelligent Beings, which in the Order of Nature, and the *Analogy of Things*, is appropriated for the *spiritual* World: But that they are endow'd with a *rational Soul*, and bodily Sense, to communicate with the *material* World, and with a *Spirit* and *divine Senses*, to communicate with the *supreme Infinite*. So that the mention'd *metaphysical Axiom* continues just and true, as it shou'd according to the *Analogy of Things*: That as *material* things are convey'd to the *rational Soul*, through the *bodily Senses*, so

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*Spiritual*

*Spiritual Things* (those that relate to *supreme Infinite*) are convey'd through the *divine Senses*, to the *Spirit*. q. e. d.

Corollary I.

HENCE, in the *Analogy of Things*, as the *Light* of the *Sun* (that noble and glorious *Representation, Image, and Vicegerent* of the *supreme Infinite*, in the *material World*) is the *Medium*, through which *material Things* are seen and perceiv'd in our *System*, so the *essential Light* of the *supreme Infinite* himself, is the sole *Medium*, by and through which, his Nature and infinite Perfections are to be understood, and comprehended: And therefore, as certainly as the *Sun* sends forth his *Light* on the whole *material World* without Bounds or Limits, on the *Just, and on the Unjust*; so certainly, the *Sun of Righteousness*, the *Pattern and Archetype* of our *material Sun*, sends forth his enlightening and enlivening Beams on all the *System* of created intelligent Beings; and is, *that Light which enlightens every Man that cometh into the World.*

Corollary II.

HENCE it is evident, that he who wou'd judge, determine, and pursue the practical  
Con-

Conclusions of these Determinations, about the Nature and Properties of *spiritual* and *divine Things*, by his *Reason*, wou'd act as incongruously, and contrary to the *Analogy of Nature*, as he who wou'd taste Colours (as such) and look into Sounds. The highest that this *Faculty* can justly and congruously pretend to in these Matters, is from the known, certain, and experienced Nature, and Properties of *material* Things, (to which the *rational Faculty* is in some measure adequate) by a proper *Analogy*: And from the *Visible's* being low *Images* of the *invisible*, and *spiritual*; to frame *similar*, but imperfect *Likenesses*, and Representations of these superior Objects, their Natures and Properties, as we have endeavoured to do in the preceeding *Propositions*; and this really, and in fact, is all that *Reason* can do in these sublime Matters.

Corollary III.

HENCE, we may discover the Errors and impieties of *Spinoza* and *Hobbes*; and the Mistakes of a *later Philosopher*, I mean (the otherwise *ingenious*) Mr. *Lock*. The *first* of these considered this *universal System of Things*, as a kind of a *Huge-brute-animal*, actuated by a fatal, necessary, unintelligent, and designing Principle; without either Wis-

dom or Choice. The *second* considered *human Nature* (not as it really is, in its present State of *Probation* and *Purification*, a Mixture of moral and natural Good and Evil, but) in its *diabolical* and *reprobated* Estate: Not as groaning under its present State of Corruption, and waiting and panting for the glorious *Liberty of the Sons and Children of God*; but as it will be in an habitual confirm'd Estate of the *Anarchy* and *Rebellion* of its Faculties one against another. In a Word, he considered *human Nature* only, as it is in the worst of Men; or as it is supposed to be at last in a State of *final Impenitence*, and harden'd *Impiety*; and this he took as his *Original* to copy after, and his *Model*, whereupon he was to frame his *human Creature*: And it must be allowed he has wrought it up to *the Life*. The *third* consider'd *Man* and his *Faculties*; not indeed in their already *reprobated* and *hellish Estate*; but as he really now is, in the World, a Composition of moral and natural Good and Evil: And this State he has very fairly and justly represented so far as it goes. But then, either having no Notion, or at least no Regard to his *higher Faculties* (which in natural and lapsed Man lye buried under the Rubbish of his present Corruptions and Sensuality;) nor to his *regenerated, re-integrated and reestablished Estate*, (to which he

he must be restored, before he can reach the *End* of his *Being*.) No, not so much as to consider Man as he really is, a *fallen, depraved, vitiated* Creature, (in which State his *lower*, his *rational* Faculties are impaired; his *higher* Faculties in some measure obliterated, at least buried and oppressed by the Load of present Corruption and Sensuality: And all of them in a State of *Anarchy*, Rebellion, and contrariety one to another.) I say, from having no Regard to those *other* different *real* Estates of *human Nature*, his Accounts of its Faculties are *unequal and imperfect*. His *Principles* when apply'd (by himself or his Disciples) to Subjects (to which Faculties are appropriated, higher than those he *elicits* out of the meerapsed State of human Nature) of a more elevated Order (such as *Christianity* and its *poly Mysteries*, *Faith*, *Grace*, *divine Revelation* and *Inspiration*, and the Means of Man's Recovery) debase these into *meer* *heathenish* *Morals*, or *human* *Philosophy*, and sink the *Oeconomy* of *the whole Wisdom* of *the Godhead*, even below the poor Contrivances and barren Speculations of many of the *gentile Sophists*. But those who fully understand the *Principles*, and are convinced of the *Truth* of the *Propositions*, I have laid down, will easily perceive the ground of the Errors and Mistakes of these three



*Setts of Philosophers*, and be able to answer their Arguments without my being obliged to detail them.

*Proposition XXIII.*

IN the *Analogy of Things*, and order of Nature, as the *material World* is to *universal Space* its highest Limit and Boundary so is the *spiritual World* to the *supreme and absolute Infinite*, the highest Limit and Boundary of all Things.

*Demonstrat.* This is evident from *Lemma* i. with *Prop.* 9. and 17. There is a beautiful *Analogy* and Uniformity running through the whole *System* of Creatures. The *Visible* and the *Created* are *Images* of the *Invisible* and of the *Increated*. The *System* of intelligent Beings are more exalted, more noble, and more immediate *Images* of the *supreme Infinite*. The *Analogy of Things* runs quite through the whole *System* of Creatures, up to their original Pattern and *Archetype* in the divine Nature, in a continued Subordination and Scale, according to their respective Natures. The *material World* is an *Image* of the *spiritual World* as the *spiritual World* is of the *supreme Infinite*. As *infinite Space* is the *Locus* and Boundary of the *material World*, so is the *supreme Infinite*, the *analogical Locus* of the whole

whom they all live, move, and have their being) and the Omega of all Things, spiritual and material. And as Space is similar to a spiritual Substance, so is that to the divine Substance, therefore, &c. q. e. d.

### Corollary I.

HENCE material and spiritual Substances are both of them extended; for since the material World is to universal Space, as the spiritual World is to the supreme Infinite; and since both Matter and Space are extended, so also must spiritual Substances be; And the divine Ubiquity, and Omnipresence, not virtually only, but substantially and essentially, makes it not unlikely that there may be, in the divine Substance, a Resemblance of Extension (so far as a Resemblance and Similitude of Substances can reach, between a relative Infinite, such as universal Space is, and the supreme Infinite) that infinitely more pure and perfect, than that of created Space is, or can be.

### Corollary II.

HENCE Matter and Spirit are Opposites in every other Quality, except in that of Extension; for since by Conversion of Ratio's, the material World is to the spiritual World,

as *infinite Space* is to the *supreme Infinite*, and since these two last, to wit, *universal Space*, and the *supreme Infinite*, are opposites in every other Quality, but in Resemblance of Substances, and that too, as an *absolutely infinite Distance*, as is evident at first View; therefore the *other two* must be opposite in every other Quality but *Extension*. For tho' *extended Matter* be divisible by being *extended*; yet *Space* is not actually to be divided, or one part of it separated from another. Since it is the *universal Locus* of, and penetrates all Bodies. And it is in *this* Sense, that the Opposition of these two Qualities in *Body* and *Spirit* is meant here.

#### Proposition XXIV.

By the *Analogy of Things*, and according to the order of Nature, a *Spirit* is an *extended, penetrable, active, indivisible, intelligent Substance*.

*Demonstrat.* By Def. 2. *Matter* is an *extended, impenetrable, passive, unintelligent, divisible Substance*: And since by the preceding Corollary, *Matter* and *Spirit* are in every other Quality opposite, except in that of *Extension*; therefore in Place of all the Qualities in the *Definition of Matter*, putting their Opposites, excepting in that of *extension*.

*extended* Substance, (for *Extension* must imply a Subject) and then a *Spirit* will become according to the *Proposition*, an *extended, penetrable, active, indivisible, intelligent* Substance. I have chosen the Word *Intelligent* in this, and its opposite *Unintelligent* in the other Definition. To wit, in that of *Body*, rather than that of *Thinking*; because *Intelligence* is the Source and Principle of *Thinking*, and expresses the whole of all the Faculties of *spiritual* Substances.

### Corollary I.

HENCE, in Respect to their Substances only, a *material* Substance is an infinitely condensed or incrassated *spiritual* Substance: And on the other Hand, a *spiritual* Substance is an infinitely rarify'd or refin'd material Substance. As we have it in Holy Writ, there is a *natural* (or material) *Body*, and *there is a spiritual* and a *glorified Body*. For since *Matter* and *Spirit* have the Foundation of their Qualities common to both, to wit, an *extended* Substance; Since all their other Qualities are the one, respectively the Opposite or Negative of the other. Since *rarifying* any Quality in *Body* and *Spirit*, is subtracting from its Intension and Energy; and therefore an infi-

nite Rarefaction of a Quality is subtracting it intirely; and there being no *mean* between *Penetrability* and *Impenetrability*, between *Passivity* and *Activity*, *Divisibility* and *Indivisibility*, *Intelligence* and *Unintelligence*, they being contrary and opposite; therefore the infinite Rarefaction of the one Quality is the Position of its contrary; for tho' the finite *Subtraction* of a *negative* Quality (admitting no *mean*) puts nothing, yet the infinite *Subtraction* of such a *negative* Quality puts the *affirmative* (thus  $-\infty \times -\infty = +\infty$ . but  $-\infty \circ + -\infty = +\infty$ ) therefore in *Matter*, *subtracting infinitely*, all its *positive* Qualities, or which is the same, (by the *preceding Corollary*) in *Matter*, *subtracting infinitely* the *negative* Qualities of *Spirit*, and then a *material* Substance will become a *spiritual* Substance; but since an infinite Rarefaction of a contrary Quality is the same with an *infinite Subtraction* of that contrary Quality, and since the same manner of Reasoning holds good in an *infinite Condensation* of the Qualities in *Spirit*: Therefore in Respect of their *Substances* only a *material* Substance, &c. q.e.d. The Case in short, as I conceive it, is thus: A *spiritual* Substance, when infinitely condensed and *incrassated*, loses its Qualities of *Penetrability*, *Activity*, *Indivisibility*, and *Intelligence*. These being lock'd up, and as it



it were *cramp'd*, in this *Condensation* of their *Substratum* (or the Substance in which they essentially inher'd,) thus *infinitely compress'd*. The *Actuality* (as the *Metaphysicians* speak) of these *spiritual* Qualities being thus shut up and imprison'd, tho' their *Potentiality* be not quite destroy'd, and thus a *crass, extended, impenetrable, passive, divisible, unintelligent* Substance is generated, which we call *Matter*; but when this *Matter* thus form'd of a *spiritual* Substance, is again *infinitely* refin'd and exalted; these *Powers* and *Qualities* are unloosed, set at freedom again, and exert themselves as formerly, and thus become what they were originally made. But we must take care not to imagine that any *finite Subtilsation, Division, Refinement, or Exaltation* of gross *Brute-matter*, can in any the least Degree, by any finite created Powers whatsoever, bring it to any but an infinitely distant Approach to this State of *Spirituality*; since it has been demonstrated, in the first Chapter of this Part, that no Finite, how great soever, can be any finite Part of any *relative Infinite* how small soever: No Power less than his, *who out of the very Stones cou'd raise* (by Virtue of their original *Potentiality*) *Children to Abraham*, can out of *material*, bring *spiritual* Substances; or on the contrary, convert these into those.

Corol-

## Corollary II.

SINCE in ascending from *material* Substances, there can be no such thing, by the *Analogy* of Things, and Order of Nature, as a *Jump* or *Leap*, from one extreme to another, without passing through the intermediate Steps; and since in *material* Things, there are Substances of all Degrees of *Density* and *Rarity*. *Earth's* more dense than *Water*, *Water* than *Air*, *Air* than *Ether*, *Ether* than *Light*. So in the *spiritual* World, there must be *Spirits* of all Degrees of *Rarity*; the one *Set* and Rank more pure and refin'd than the other in a perpetual *Scale*, 'till they ascend so near the *supreme Infinite*, as Creatures can approach their *Creator*, or *Finites* the *absolute Infinite*. And by the same *Analogy of Things*, as in the *material* World, these several Orders of Bodies, *Earth*, *Water*, *Air*, *Ether* and *Light*, have their proper *Places*, *Elements*, and *Centers* where they rest, and whither they tend, and out of which they cannot be detain'd but by *Violence*; so in the *spiritual* World there are *Centers*, *Spheres* and *Elements* of several Orders of *Spirits*, the one more pure and refin'd than the other, (the more pure still penetrating the less pure,) where they rest and continue, to which,  
by

by their *specifick* Degree of Purity they are confin'd, out of which they cannot be detain'd but by *Violence*; all in a Subordination one to another, depending on their particular Degrees of Purity, penetrating one another without Confusion or Contrariety, 'till they arrive as near as possible to the *supreme Infinite*, who penetrates the whole *System* of Creatures.

Corollary III.

SINCE by *Prop. 15.* the *Desires* are the *cardinal* Faculty of intelligent Beings, infinitely *active* and powerful, belonging to that Principle in them, which is appropriated to the *spiritual* World. By these *Desires* therefore, those of the same *Element*, and same Degree of Purity, are enabled to communicate one with another: But since the more pure penetrate the less pure; but not *vice versa* (as is plain from *Corollary 1.* of the *preceding Proposition*;) the more pure may penetrate the less pure, contrary to their *Desires*; but not *vice versa*, this Superiority of the more pure, over the less pure, being a necessary Consequence of the greater Degree of Purity and Perfection. For as in the *material* World, the *Sun* purifies and rarifies terrestrial Bodies, the nearer they approach, or are brought to him: And at

last converts them into his Substance. So in the *spiritual* World, the *Sun's* Pattern and *Archetype*, the *Sun of Righteousness* renders those *Spirits* the more pure, penetrating (and as it were *deifies* their intimate Substances) whose *Elements* or Region is nearest him; by which they more nearly partake of his Nature, who penetrates the whole *System* of Beings.

#### Corollary IV.

HENCE, since the *supreme Spirit* in compounded intelligent Beings is more pure (as being the highest Principle, and that *Breath of Life*, which they had immediately from the divine Substance) than the *rational Soul*; the *first* penetrates the *latter*, and the *latter* is but the *Medium*, and is it self but of an intermediate Nature, between the two *Extremes*, *Body* and *Spirit*, coupling them together by its intermediate Substance: And in their primitive Order and Institution, they were in a due Subordination one to another: The *Body* to the *rational Soul*, and *both* to the *supreme Spirit*; and in this Subordination, preserv'd and maintain'd, and diligently cultivated, according to their respective Dignity; in the *Wills* obeying and approving of, and in the *whole compound's* pursuing the practical Inferences deduced

duced in this *State*, did the *original Rectitude* of these compounded intelligent Beings consist. And in the Contrariety, Contradiction and Rebellion of these different *Principles* one against another, in the *Will's* obeying and approving of, and the *whole Compound's* pursuing the practical Conclusions deduced under this State of *Anarchy*, does the *Fall, Lapse, and Degeneracy* of this *Set* of intelligent Beings consist. And for the restoring, rectifying and re-establishing the primitive Order and State of these three Principles, to an habitual and lasting Subordination, was the Incarnation of the *divine* LOGOS, and whole *Oeconomy* of the Redemption of Man.

Corollary V.

HENCE, the *supreme Spirit* may be dark, dead, and almost quite obliterated, as to its *ouvert* Acts (the *Principle* it self being essential to, and interwoven with the most intimate Natures of all intelligent Beings,) when the *rational Soul* is full of *Ideas*, Pictures and Images of Things. And on the other Hand, the *supreme Spirit* may be full of *Light, Brightness, substantial Knowledge, Joy and Peace*, when the *rational Soul* is but weak, faint and languid, and almost void of all *Ideas* and Images; these being  
not



not only separable, but at last to be *actual-ly* separated. (So far at least as the Acts of the *rational Soul* can affect the *supreme Spirit*,) when the LOGOS (that *eternal and essential Word of God*) which is *quick and powerful, and sharper than a two-edged Sword shall pierce, to the dividing asunder the Soul and Spirit.*

### SCHOLIUM GENERALE.

THUS I have endeavoured to give some faint and imperfect Images of the highest and most sublime Speculations of *Religion* and its *Philosophy*, in the *preceding Propositions* and *Corollaries*. And tho' I am very far from thinking they are even just and compleat Images, and such as might be draw from the same Principles, by a more skilful Hand: Yet I am fully convinced, the *Propositions* and *Corollaries* themselves are true and just, as to their Substance, whatever may be in my Way of explaining or demonstrating them; so far at least, as *Reason* can find out the *Truth* in such *sublime Mysteries*. I am also well satisfied that *Reason* can with any Propriety or Justness apply itself to Objects, to which it is not *adequate* and appropriated, after no other Manner, but by supposing those Objects, to which it is *adequate, Images*

ges or Representations of those other Objects, to which it is not adequate; there being no other possible way for *Reason* to find a *Medium* of its Knowledge of Objects, that are convey'd to the rational Understanding, by none of the bodily Senses, (as the proper Objects of its Faculties are.) And there being an absolute Necessity from the *Simplicity* and *Uniformity* of the *divine Nature*, and of his manner of Operating, that all his Works shou'd be Resemblances and *Images* one of another, (more or less perfect, according to their respective Natures) and also of himself, their *original Pattern* and *Architype*. This manner of Reasoning, and this *Medium* of rational Knowledge, duly instituted, must be just and true, as far as it reaches. And thus far *Reason* can go, and not one step further in the Knowledge of superior Objects; it can frame and form *Images* of these superior Objects, from what it finds and certainly knows of the *material* World, to which it is in some measure adequate. *Images* I say, not *metaphorical* only but *real* and *physical*, as a Statue represents a Man, a Picture in *Miniature*, one from the Life, as a *Seed* (which is really the Plant it self in little) does a grown Plant, or an *Embryo*, the adult Animal: This is the Boundary of *Reason* in these superior Objects. And it is very  
obser-

observable, that there are various Images in Nature, and in the *intellectual Species* of Things (fram'd upon what the Senses have already convey'd into the rational Understanding) of all the most unconceivable, the most abstruse and sublime *Mysteries* of Religion and its Philosophy ; each superior to another. Can there be a more perfect, noble, or lively Image in this lower World, of the *divine Nature, Light, Benignity, Greatness, and Power* required, than that of the *Sun*, in respect of our *planetary System*. His Beams shine, and are transmitted through all the *planetary* and *cometary* Regions, even into the *Systems* of the *fix'd Stars*. He *attracts* all the *Planets* and *Comets* in our *System*, and is the Source of all their regular, uniform, and constant Motions and Influences. He warms, cheers, enlivens, and fertilises all the *Elements, Vegetables, and Animals* ; and is indeed the *material Deity* of this inferior World. Is there not a plain and obvious Image of the EVER-BLESSED-TRINITY IN UNITY, in every Order of Creatures ? In the three Dimensions of Bodies ? In Nature's never rising above the *third* Dimension in her regular Operations, as was shown in *Scholium Prop. 12* ? In the *three* infinite Powers of *universal Space* ? In the *three* generic Divisions of Objects ? *Matter, created Spirit,*

*Spirit*, and the *supreme Infinite* in the *three* Distinctions and *universal Principles* that comprehend the whole of material intelligent Beings? The *Faculty*, *Object*, and the *Congruity* or *Incongruity* between these? The *Pro-fane* and *Ignorant* may make a jest of this *ternary Chain*, and ascribe it to *Chance* or *Fortune*. But the *Analogy of things*, and the regular *Uniformity* in *Nature*, make it evident to a *Demonstration*, that it must have had its *Rise* in its *original Pattern* and *Archetype*, the *divine Nature*. Even the eternal *Generation* of the *second Principle* in the *Godhead*, of the *first*, and the eternal *Pro-cession* of the *third Principle* from the *first* and *second*, comes evidently out of this *an-logical Ternary*, when elevated to its *Ori-in* in the *divine Nature*; as was shewn in *Scholium 2. Prop. 17*. And the *Impossibility* of increasing or multiplying the *divine* and *supreme Infinitude*, even by it self; so far as that it is incapable of *Increase*, or *Diminution*; as was shewn in *Corollary 2. Prop. 12*. Pictures forth the *Unity* of the *divine Na-ture*, in these *three Relations* of the *whole* to the *whole*. How noble a Representation in created Things, is the *universal Space* of the *divine Ubiquity*, *Infinitude*, and *spiri-tual Nature*? How lively a Picture in the *in-lectual Species* of things, of *Creation*, or of *Gods producing the things that be*, out of

the things that were not, is that *Proposition* demonstrated *Corol.*

2. *Prop.* 2.

$$\frac{1+1+1+1+1+1}{1} \text{ &c.}$$

$$x0 = \infty \quad x0 = 1. \text{ or } \odot + \odot + \odot + \odot + \odot \text{ &c. } x$$

$$0 = \infty \quad \odot \times 0 = \odot ?$$

The Production of a *Plant* from its *Miniature* in the Seed; and of an *Animal* from an *Animalcule*, is an astonishing Representation of the Resurrection of the Body. These and many such Resemblances, and Images in Nature, in the *sensible* and *visible* things, and in the *intellectual Species* of things, derived through the Senses; might be brought to illustrate and confirm the greatest Difficulties and most abstruse Mysteries of *Religion*, and its *Philosophy*. So certain and universal is the *beautiful Analogy of Things*, and so careful has the *kind and bountiful Author* of our Beings been, to supply us with Evidences in our *lower Faculties*, and *lapsed Estate*, of *those Truths* he requires us to believe and receive. The *full and compleat Conception* and Knowledge of which belong not but to our *superior Faculties*, and to our *restored and re-established Estate*. May we then use the Assistances, *Reason*, and the *divine Bounty*, has afforded us, (I may almost say, even beyond and out of their natural Order) for increasing our *Faith*, cultivating and expanding our *superior Faculties*, rescuing us out of our *Degeneracy* and

Corrup



Corruption, into the glorious Liberty of the Sons and Children of God, and then, in his Light we shall see Light, Amen.

### CHAP. III.

#### Of the USE of the Arithmetick for Infinites.

§ I. **I**N arithmetical Progression ; in which let  $a$  denote the first,  $v$  the last term,  $x$  the Difference,  $t$  the Number of Terms, and  $z$  the Sum of all the Terms, which in an ascending Progression are  $a, a + x, a + 2x, a + 3x, \&c.$  but in a descending Progression  $a, a - x, a - 2x, a - 3x, \&c.$

Having any three of these five  $a, v, x, t, z$ . you may find the other two by help of these two Lemma's.

Lem. 1.  $\{ v = a + tx - x \}$  but  $v = a - tx + x$ .

Lem. 2.  $\{ 2z = t \times a + x \}$  when it descends.

I shall here only treat of ascending Progressions, judging those that descend as not properly belonging to the Arithmetick of Infinites.

Now in ascending Progressions both  $t$  and  $v$  are infinite.

Therefore Corol. 1.  $\{ v = tx \}$  in the Case of

And Corol. 2.  $\{ 2z = tv \}$  Infinites.

K 2

Because

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Because Infinites are not all equal, but vary in their Ratio's to one another, as much as finite Quantities do; therefore to avoid Confusion, I suppose  $v = \infty \times 1 = 1 + 1 + 1 + 1 + 1 + 1 \text{ \&c.}$

So from *Corol. 1.*  $t = \frac{\infty}{x} = \infty \times \frac{1}{x} = \frac{1}{x} + \frac{1}{x} + \frac{1}{x} + \frac{1}{x} + \frac{1}{x} \text{ \&c.}$

*Prop. 1*  $z = \frac{\infty^2}{2x}$ . This follows from substituting the Value of  $t$ , viz.  $\frac{\infty}{x}$  in *Corol. 2.*

*Examp. 1.* Let  $x = 1$ . then  $1 + 2 + 3 + 4 + 5 \text{ \&c.} = z = \frac{\infty^2}{2}$ , that is, the Sum of all

the natural Numbers continued in *infinitum* is equal to half the Square of Infinite: And here it is to be noted, that in this Case only  $t = v = \infty \times 1 = 1 + 1 + 1 + 1 + 1 \text{ \&c.}$

*Examp. 2.* Let  $x = \frac{1}{2}$ , then  $1 + 1\frac{1}{2} + 2 + 2\frac{1}{2} + 3 + 3\frac{1}{2} \text{ \&c.} = z = \infty^2 =$  to the Square of Infinite.

And thus may you find the Ratio between the Sum of any ascending arithmetical Progression, and the Square of Infinite; and note that by infinite, is always understood Infinite in

in General, or Infinite of the simplest Nature and lowest Degree,  $1+1+1+1$ , &c. unless it be otherwise expressly declared.

*Corol.*  $\infty + \frac{1}{\sqrt{2x}}$  is the Root of a Square,

which Square is equal to the Sum of any arithmetical Progression ascending in infinitum.

*Examp.* Let  $=x2$ , then  $\infty \times \frac{1}{2}$  (or an infinite number of  $\frac{1}{2}$ ) is the Root of a Square equal to the Sum of that Progression, viz.  $1+3+5+7$  &c.  $= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$  &c.  $|^2$ .

*Corol.* 2. Let  $a = \infty n$ ,  $v = \infty m$ .  $t = \infty p$ . These values of  $a$ ,  $v$ ,  $t$ , being substituted in Lem. 2. will give you  $2z = \infty p \times \infty n + \infty m$   
 $= \infty^2 \times pn + pm$ , or  $z = \frac{\infty^2 \times pn + pm}{2}$ , and

the common Difference  $x = \frac{m-n}{p}$ , as you may easily find from Lem. 1.

You may express the Sum  $z$  otherwise, for

$t = \frac{v-a}{x}$  by Lem. 1. Therefore  $\frac{v-a}{x} \times$   
 $v+a = 2z$  by Lemma 2d that is  $z = \frac{v^2 - a^2}{2x}$

$= \frac{\infty^2 m^2 - \infty^2 n^2}{2x} = \frac{\infty^2 \times mm - nn}{2x}$ ,

*Corol. 3.* From *Prop. 1.* and *Corol. 2.* it is evident that in two arithmetical Progressions having the same common Difference  $x$ , the Sum of that whose Terms are infinite, is to the Sum of that, whose Terms are finite, as  $m^2 - n^2$  to 1. For the Sum of the first, by *Corol. 2.* is  $\infty^2 \times m^2 - n^2$

—,  $x$  the Sum of the other, by *Prop. 1.*

$2x$   
is  $\infty^2 \times \frac{1}{2} x$ .

*Corol. 4.* From hence may be solved this Problem. Any arithmetical Progression being given as  $a, a+x, a+2x, a+3x, \&c.$  whose terms are finite; to find another consisting of infinite Terms, that shall have the same Difference  $x$ , and whose Sum shall be equal to the Sum of the given Progression. *Solution.*  $m^2 - n^2 = 1$  from *Corol. 3.* *Ex hypothesi.* Therefore  $m^2 = 1 + n^2$ ; so to find  $m$  and  $n$ , is an indetermin'd Problem, whose Solution

by the known Methods  $m =$  will give you  $\frac{e^2 + 1}{e^2 - 1}$ ,

$$n = \frac{2e}{e^2 - 1},$$

where  $e$  is any Number ( $\Delta 1$ ) taken at Pleasure. So then these Values of  $m$  and  $n$  being substituted in the Values of  $a$  and  $v$ , of *Corol. 2.* will give you the first Term  $a$

$= \infty \times \frac{2e}{e^2 - 1}$ , and the last  $v = \infty \times \frac{e^2 + 1}{e^2 - 1}$

and calling the common Difference  $x$ , you will

will have a Progression, each of whose Terms is infinite, and whose Sum shall be equal to the Sum of any other arithmetical Progression, whose first Term is finite, and the common Difference  $x$ ; and because  $e$  is an arbitrary Number, therefore you have as many of such Progressions as you please, to answer the Problem.

*Examp.* Let the given Progression be  $1 + 2 + 3 + 4 + 5$  &c. where  $x = 1$ , so the Sum thereof is  $= \infty^2 \times \frac{1}{2}$  (as in *Ex. 1. Prop. 1*) now

taking  $e=2$ , you will have  $n = \frac{e^2 - 1}{2e} = \frac{4}{3}$  &c.  
 $m = e^2 + 1$

$= \frac{5}{3}$ , which from the second Expression  
 $= \frac{1}{3}$

of *Corol. 2.* will give the Sum of the Series  
 $(\infty \times \frac{4}{3}, 1 + \infty \frac{4}{3}, 2 + \infty \frac{4}{3}, 3 + \infty \frac{4}{3}, \&c.)$   
 $m^2 - n^2$

$\infty^2 \times \frac{1}{2} = \infty^2 \times \frac{1}{2}$ , which is  
 $\frac{1}{2} x$

the same with that of the given Progression consisting of finite Terms.

*Schol.* The Problem may be made more general, and the Solution as easy, if it were required, that the Sum of the Series, consisting of infinite Terms, should be to that of the other consisting of finite Terms in any given Ratio of  $r^2$  to  $s^2$ , for then it must be to find  $m$  and  $n$ , such that  $m^2 - n^2, 1 :: r^2 : s^2$ .

K 4

*Prop.*



*Prop. 2.* Let all Things be as in *Prop. 1.* except the last Term, which here we shall suppose  $v =$

$$\infty n, \text{ in which Case I say } \left\{ z = \infty^2 \times \frac{n n}{2 x} \right\}$$

*Examp. 1.* Let  $n=2, x=1; a=1$ , so the Progression will be 1, 2, 3, 4, 5, &c. till the last Term be  $\infty 2$ , I say the Sum  $1+2+3+4+5$ , &c.  $= z = \infty^2 \times 2 =$  double the Square of Infinite.

§. II. IN geometrical Progression, let  $a$  denote the first, and  $v$  the last Term, the Ratio of the Terms, that of  $r$  to  $s$ ,  $t$  the Number, and  $z$  the Sum of all the Terms of any geometrical Progression, which will be  $a$ ,

$$\frac{s a}{r}, \frac{s^2 a}{r^2}, \frac{s^3 a}{r^3}, \frac{s^4 a}{r^4},$$

Of these five Things, *viz.* the first Term  $a$ ,  
the last Term  $v$ , the Ratio of the Terms  $\frac{s}{r}$ , the  
number  $t$ , and the Sum of the Terms  $z$ , having any three, you may find the other two by these two *Lemma's*.

$$\left. \begin{aligned} \text{Lem. 1. } v &= \frac{s^t a}{r^{t-1}} \times a \\ \text{Lem. 2. } s z + r a &= r z + v \end{aligned} \right\}$$

From

From *Lemma 2d.* it follows that  $z = \frac{ra - sv}{r - s}$   
 when the Progression ascends, that is, when  $r < s$ .

But when the Progression descends, that is, when  $r > s$ , then from *Lemma II.* it will be  $z = \frac{sv - ra}{s - r}$

Case 1. In descending Progressions where the Number of Terms are infinite, the last Term  $v$  will be  $= 0$ .

*Prop. 1.* In all descending Progressions, whose Number of Terms are infinite, and first Term finite, the Sum of the Progression is a finite quantity, viz.  $z = \frac{ra}{r - s}$

*Corol. 1.* Let  $a = r$ , then  $z = \frac{r^2}{r - s}$

*Examp.* Let  $r = 1$ ,  $s = \frac{1}{2}$ , then  $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \text{ \&c.} = 2 = \frac{1}{r - s} = z$

*Prob.* Having the first Term  $a$ , and the Ratio of the Terms  $\frac{r}{s}$  of any geometrical Progression descending in *infinitum*, till  $v = 0$  to find another Progression descending in *infinitum*, whose first Term shall be any given number

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ber  $b$ , and its Sum equal to the Sum of the given Progression.

*Sol.* Let  $m$  to  $n$  be the Ratio of the Terms  
of the Progression sought; now  $\frac{r a}{r - s}$  is the  
Sum of the given Progression, by *Prop.* I. and  
for the same reason  $\frac{m b}{m - n}$  must be the Sum of the  
Progression sought, therefore from the Condi-  
tion of the Problem fought  $\frac{m b}{m - n} = \frac{r a}{r - s}$ ,

which gives  $n = \frac{(m \times r a - r b + s b)}{r a}$ , where  $a, r, s, b$

are given, and  $m$  may be taken at Pleasure; and  
so you have  $n$  from this Equation; and conse-  
quently the Progression sought is found; for  
a Progression is found, when you have the first  
Term  $b$ , and the Ratio of the Terms  $m$  to  $n$ .

*Example.* Let  $r = 1, s = \frac{1}{2}, a = 1; b = \frac{2}{3}$ ,  
I assume  $m = \frac{2}{3} (= b)$  so the Equation will give  
 $n = \frac{4}{3}$ , therefore  $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \text{ \&c.} = \frac{2}{3} + \frac{4}{9} + \frac{8}{27} \text{ \&c.}$

And here it is to be noted that since  $m$  is  
taken at Pleasure, the Problem is indetermin-  
ed, and so you may find as many Progressi-  
ons as you please, whose Sum shall be equal  
to the Sum of any given Progression.

*Schol.*

*Schol.* The Problem will be more general if it be required that the Sum of the given Pro-

gression  $\frac{r a}{r-s}$  be to the Sum of the Progressi-

on sought  $\frac{m b}{m-n}$  in any given Ratio  $b$  to  $k$  viz.

$$\frac{r a}{r-s} : \frac{m b}{m-n} :: b : k, \text{ which gives } z = \frac{m \times a r k + b b s - b b r}{a k r}$$

; where  $m$  is at Pleasure.

Case II. In ascending Progressions, whose number of Terms are infinite, the last Term  $v$  will be  $= \infty \times e$ ; but first I shall consider these Progressions in which  $e = 1$ , that is, where the last Term is an infinite Number of Units.

*Prop. 2.*  $z = \infty \times \frac{s}{s-r} (= \frac{s v}{s-r})$  for  $r a$  vanishes, because  $a$  is infinite, and  $v$  infinite.

But here it is carefully to be observed, that in all these Progressions it is necessary to make  $r = r$ , so that  $r$  will be the first and  $s$  the second Term of the Progression; and indeed in all geometrical Progressions, there is a Convenience of expressing the Ratio of the Terms by the two first Terms of the Progression.

This

This being premised, it is evident that  $\frac{s}{s-r}$  is the infinitely small and equal part, of which an infinite Number gives the Sum of any geometrical Progression ascending in *infinitum*.

*Examp. 1.* Let  $r=1$ ,  $s=3$ , then  $\frac{s}{s-r} = \frac{3}{3-1} = \frac{3}{2}$ , which shews  $z = \infty \times \frac{3}{2}$ , that is  $1+3+9+27+81 \text{ \&c.} = 3 + \frac{3}{2} + \frac{3}{2} + \frac{3}{2} \text{ \&c.}$

*Examp. 2.* Let  $r=1$ ,  $s=\frac{3}{2}$ , so  $\frac{s}{s-r} = \frac{\frac{3}{2}}{\frac{3}{2}-1} = 3$ ,  
Ergo  $z = \infty \times 3$ , that is,  $1 + \frac{9}{4} + \frac{27}{8} + \frac{81}{16} \text{ \&c.} = 3+3+3+3 \text{ \&c.}$

*Problem.* To find a Progression, which shall have any given Number ( $a$ ) for the equal Part of which its Sum consists. *Solution.* Let  $r$  be the first, and  $s$  the second Term of the Pro-

gression sought, then (because  $\frac{s}{s-r}$  is the e-

qual Part for every Progression)  $\frac{s}{s-r} = a$   
which gives  $s =$

$\frac{r a}{a-1}$ , so taking the first Term  $r$  at Pleasure,

the second shall be  $\frac{r a}{a-1}$ .

*Examp.*



*Examp.* To find a Progression whose Sum shall be  $= \infty \times 2$ . In this Case  $a = 2$ , so ta-

king  $r = 1$ , it will give  $\frac{ra}{a-1} = 2$  which shews

that the Sum of a Progression, whose first Term is 1, and second Term is  $2 = \infty 2$ , or  $1 + 2 + 4 + 8 \text{ \&c.} = 2 + 2 + 2 + 2 \text{ \&c.}$

*Scholium.* Because  $r$  is taken at Pleasure, therefore you may find as many Progressions as you please, whose Sums shall be all equal, because each of them is  $= \infty \times a$ .

*Prob. 2.* Having any Progression ascending, to find another whose Sum shall be equal to the Sum of the given one. Let  $r$  be the first,  $s$  the second Term of the given Progression; and  $m$  the first,  $n$  the second Term of

the Progression sought, then  $\infty \times \frac{s}{s-r}$  is the

Sum of the given one, and  $\infty \times \frac{m}{m-n}$  is the

Sum of the Progression sought, Ergo  $\infty \times \frac{s}{s-r} = \infty \times \frac{m}{m-n}$ ,

which gives  $n = \frac{rm}{s}$ ,  $m$  may

be taken at Pleasure, and so you have  $n$ .

*Schol.*

*Schol.* So may you find a Progression, whose Sum shall be to the Sum of the given one, in

any given Ratio  $b$  to  $k$ , for then — : — ::

$$b:k: \text{so that } n = \frac{m \times s \times b \times r \times k - s k}{s b}$$

§. III. Before I proceed to other Progressions it will be necessary to subjoin some Things, which were omitted in § I. concerning arithmetical Progression, and which should have immediately preceeded § 2.

*Lemma.*  $z = \infty^2 \times \frac{1}{2x}$ , when  $v = \infty \times 1$ . by

*Prop.* 1. of § 1.

*Prob.* Having the common Difference  $x$ , and the last Term  $v = \infty \times 1$  of any arithmetical Progression ascending in *infinitum*; to find another arithmetical Progression, whose Sum shall be equal to the Sum of the given one.

Let  $e$  be the common Difference,  $\infty \times n$  the last, and  $y$  the Sum of all the Terms of the

Progression sought; then  $y = \infty^2 \times \frac{n}{2e}$  : by

*Prop.* 2. of § 1. Now because  $z = \infty^2 \times \frac{1}{2x}$  is the

Sum of the given Progression, therefore from the

the Condition of the Problem  $\frac{1}{2x} = \frac{nn}{2e}$ , so  $e = x$

$nn$ ; you may take  $n$  at Pleasure, and so you have  $e$  and consequently the Progression sought.

*Examp.* 1. Let the given Progression be 1, 2, 3, 4, 5, &c. to  $\infty + 1$ , where  $x = 1$ , so  $e = n$ ; calling  $n = 2$ , you have  $e = 4$  for the common Difference of the Progression sought, viz, 1, 5, 9, 13, &c. to  $\infty \times 2$ . I say then that the Sums of these two Progressions are equal.

*Corol.*  $n = \frac{\sqrt{e}}{\sqrt{x}}$  so taking  $e$  at pleasure you have  $n$ , as in the former Example, if you call  $e$ , 2, then  $n = \sqrt{2}$ , so the Progression sought is, 1, 3, 5, 7, &c. to  $\infty \times \sqrt{2}$ , whose Sum shall be equal to  $1 + 2 + 3 + 5 + 6$  &c. to  $\infty \times 1$ . So that in solving this Problem, you may either make the common Difference, or the infinitely small Part ( $n$ ) of the last Term, what you please; and because one of the two may be taken at Pleasure, therefore the Problem is indetermined, and consequently you may find as many Progressions as you please, whose Sums shall each of them be equal to the Sum of the given Progression.

*Scholium.* in the same Manner you may find as many Progressions as you please, the Sum of each of which shall be to the Sum of the given one in any given Ratio of  $k$  to  $b$ ; for then

$nn$

$\frac{n}{2} : k :: e : b$ , which gives  $ke = bx n^2$ ; so that of these two ( $e, n$ ) taking one at Pleasure, you have the other from this Equation.

*Problem. 2.* Having the Sum  $\infty^2 \times q$  of any Progression, to find another that shall have a given common Difference  $e$ , and whose Sum shall be equal to the given Sum. Let  $\infty \times n$  be the last Term of the Progression sought, then

its Sum will be  $= \infty^2 \times \frac{n^2}{2e}$ ; therefore by the

Condition of the Problem  $\frac{n^2}{2e} = q$ . so that  $n =$

$\sqrt{2eq}$ : but,  $e$  and  $q$  are given, therefore  $n$  is known.

*Examp.* Let  $q = 1$ , it is requir'd to find a Progression whose Difference shall be 1, and its Sum  $= \infty^2 \times 1$  or the Square of infinite; now because  $q = 1$ , and  $e = 1$ , therefore  $n = \sqrt{2}$ , so that  $\infty \times \sqrt{2}$  shall be the last Term of a Progression 1, 2, 3, 4, 5, 6, 7, &c. whose Sum shall be  $= \infty^2$ ; but by *Examp. 2d.* of *Prop 1.* of § 1. if  $x = \frac{1}{2}$ , and  $v \infty$ , then the Sum  $z = \infty^2$ , Ergo  $1 + 2 + 3 + 4 + 5$  &c. to  $\infty \times \sqrt{2} = 1 + 1\frac{1}{2} + 2 + 2\frac{1}{2}$  &c. to  $\infty \times 1$ .

*Examp. 2.* Let  $q = \frac{1}{2}$ , and  $e = 2$ , then  $n = \sqrt{2}$ , so that a Progression whose common Difference

$$I \quad ex e + I$$

$$\S \text{ V. Lemma } == e = 1 + er + \frac{1 - r^1}{1} + \frac{2}{1} \times r^2 + e \times \frac{2}{1} \times r^3 + e \times \frac{2}{1} \times r^4 + e \times \frac{2}{1} \times r^5$$

Let us now consider the various Progressions that will arise out of this Series, and what relation they have to infinite in general or  $\infty \times 1$ , and that I may proceed distinctly, I will resolve it into several Cases, beginning at the simplest, where  $r=1$ , and so proceed gradually to  $r=2$ ,  $r=3$ , &c. And where the Series is neither in an arithmetical nor yet in a geometrical Progression, I shall endeavour to discover according to what increase the Progression goes on; tho' in general that is plain enough from the *Lemma* of § 5.

CASE I. Let  $r=1$ , which contains an infinite Number of other Cases, according to the different Values of  $e$ , the Progression whereof I shall shew in the following Articles.

Artic. I. ( $e=1$ )  $\frac{1}{1-1} = 1+1+1+1 \text{ \&c.}$   
 $= \infty \times 1$ . PART II. L Artic.



$$\text{Artic. 2. } (e=2) \overset{\text{I}}{=} 2 = 1 + 2 + 3 + \dots$$

$$4, \text{ Sc. } = \infty^2$$

$$\text{Artic. 3. } (e=3) \overset{\text{I}}{=} 3 = 1 + 3 + 6 + \dots$$

$$10 \text{ Sc. } = \infty^3$$

$$\text{Artic. 4. } (e=4) \overset{\text{I}}{=} 4 = 1 + 4 + 10 + 20 \text{ Sc. } = \infty^4$$

Now you are to observe that the Terms of any one of these Progressions are made up of the Sum of the Terms of the Progression next preceding; for Instance, the third Term of the Progression  $\infty^3$  viz. 6, is the Sum of the three first Terms of the Progression  $= \infty^2$ ; in like Manner the fourth Term (20) of Progression in Article fourth is the Sum of the four first Terms in the Progression of Art. 3d. and the 7th Term of Art. 3d. is the Sum of the seven first Terms in Art. 2d. It is to be observed likewise, that the Terms of Art. 3d. are *triangular Numbers*, since they are the Sums of the natural Numbers of Art. 2d. and consequently the Sum of the *triangular* Progression continued in *infinitum*, is equal to the *Cube of Infinite*. Case 2d.  $r=2$ .

Art.

*Art. 1.*  $e = 1$ , then  $\frac{1}{1-2} = 1 + 2 + 4 + 8, \text{ \&c.} = \infty \times 2.$

*Art. 2.*  $e = 2$ , then  $\frac{1}{1-2} 2 = 1 + 4 + 12 + 32, \text{ \&c.} = \infty^2 \times 4.$

*Art. 3.*  $e = 3$ , then  $\frac{1}{1-2} 3 = 1 + 6 + 24 + 80, \text{ \&c.} = \infty^3 \times 8.$

*Art. 4.*  $e = 4$ , then  $\frac{1}{1-2} 4 = 1 + 8 + 40 + 160, \text{ \&c.} = \infty^4 \times 16.$

*Note,* That the Terms of *Art. 1st.* are in a geometrical Progression in the Ratio of 1 to 2.

AND it is observable, that the Terms of any following *Article* are made by the Multiplication of the Terms of *Art. 1st.* into the respective Terms of that *Article* in *Case 1.* where  $e$  has the same Value. For Instance, the third Term of *Article 2* is the Product of the third Term of *Artic. 1.* into the third Term of *Artic. 2,* of *Case 1.* and the fourth Term (*viz.* 160) is the Product of the fourth Term (*viz.* 8.) of *Artic. 2.* into the 4th Term (*viz.* 20) of *Artic. 4th. Case 1st.* And universally, let  $A$  denote any Term in any *Artic.* of *Case 1.*  $B$  any Term in *Artic. 1st* of *Case 2d.* And  $C$  any Term in any of the following *Articles* of

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*Case 2d.* I say  $C = AB$ , taking  $ABC$  in the same Order *i. e.* if  $C$  be the 5<sup>th</sup> Term, then  $A$  and  $B$  must also be the 5<sup>th</sup> Terms of their Progressions, and whatever Value ( $e$ ) has in the Progression of which  $C$  is the Term, it must have the same in that Progression where  $A$  is the Term. *Examp.* To find the 7<sup>th</sup> Term of *Artic.* 3. of *Case* 2. Here  $C$  denotes the 7<sup>th</sup> Term of a Progression in which  $e = 3$ : This shews that  $A$  is the 7<sup>th</sup> Term of *Artic.* 3. *Case* 1<sup>st</sup>, which multiplied into the 7<sup>th</sup> Term  $B$  of *Artic.* 1, *Case* 2<sup>d</sup> will give  $C$  the Term sought.

*Case* 3<sup>d</sup>.  $r = 3$

$$\text{Art. 1. } e = 1, \text{ then } \frac{1}{1-3} = 1 + 3 + 9 + 27$$

$$\text{Ec.} = \infty \times \frac{3}{2}.$$

$$\text{Art. 2. } e = 2, \text{ then } \frac{1}{1-3} \Big| 2 = 1 + 6 + 27 + 108, \text{ Ec.} = \infty^2 \times \frac{9}{4}.$$

$$\text{Art. 3. } e = 3, \text{ then } \frac{1}{1-3} \Big| 3 = 1 + 9 + 54 + 270, \text{ Ec.} = \infty^3 \times \frac{27}{8}.$$

$$\text{Art. 4. } e = 4, \text{ then } \frac{1}{1-3} \Big| 4 = 1 + 12 + 90 + 540, \text{ Ec.} = \infty^4 \times \frac{81}{16}.$$

*Note,* the first Series is a geometrical Progression going on in the Ratio of 1 to 3. And the

the following Progressions are made out of this first, with the respective ones of those in *Case 1st* in all Respects, as those of *Case 2d* already explain'd: For let *A* be any Term in any *Artic.* of *Case 1.* *B* a Term of the same Order (with *A*) in *Artic. 1.* *Case 3.* and *C* a Term of the same Order in any of the *Articles* of *Case 3d.* I say  $C = AB$ , where *e* has the same Value in *A* that it has in *C*. And so it is for all the succeeding Cases in *infinitum*: The first Series of any Case, *r* (*viz.* where  $e = 1$ ) is always a geometrical Progression in the Ratio of 1 to *r*; the following Progressions are made by the Multiplication of the Terms of this first Series into the respective Terms of those in *Case 1.* as has been shewn in *Case 1* and *2d.*

*Scholium.* To have the first Progression of

every Case, let  $e = 1$ , then  $\frac{1}{1-r} = 1 + r + r^2 + r^3 + r^4 + r^5 \text{ \&c.}$

But  $1 + r + r^2 + r^3 + r^4 \text{ \&c.} = \infty \times \frac{r}{r-1} =$

$\frac{r}{r-1} + \frac{r}{r-1} + \frac{r}{r-1} + \frac{r}{r-1} \text{ \&c.}$

BEFORE I conclude this Subject, it will be necessary to remove a very obvious and material Objection, *viz.* how it comes to pass that the Sum of the natural Numbers 1 + 2  
L 3 + 3

$+3+4+5$ , &c. should be equal to  $\frac{\infty^2}{2}$  or half the Square of Infinite, as it appears by *Ex. 1.* of *Prop. 1.* of § 1. And yet the same Sum is  $=\infty^2$  or the whole Square of Infinite, as appears by *Art. 2d.* of *Case 1.* of the last *Lemma*. This seeming Contradiction may be reconciled, if we suppose  $\infty \times 1$  to be the last Term in the Progression of *Ex. 1.* *Prop. 1.* § 1. which will make the Sum  $\frac{\infty^2}{2}$ ; and  $\infty \times \sqrt{2}$  to be the last Term in the Progression of *Art. 2d.* of *Case 1.* of the last *Lemma*; for that will make the Sum of the Progression  $=\infty^2$ , as appears by *Ex. 1.* of *Prob. 2.* § 3. So that tho' it be the same Progression going on in *infinitum*, yet the one goes on to a greater Infinite, *viz.*  $\infty \times \sqrt{2}$  than the other which ends at  $\infty \times 1$ .

BUT the plain way of reconciling the Matter depends on the common Rules of Multiplica-

$$\text{tion. } 1+2+3+4+5 \text{ \&c. } = \frac{1}{1-1} 2 = \frac{1}{1-1}$$

$$\times \frac{1}{1-1}, \text{ but } \frac{1}{1-1} = 1+1+1+1 \text{ \&c. and}$$

$$\text{therefore } 1+2+3+4+5 \text{ \&c. } = 1+1+1+1+1 \text{ \&c. } \times 1+1+1+1+1 \text{ \&c.}$$

LET us now make an actual Multiplication of six Terms only, thus

$$1+$$



$$1 + 1 + 1 + 1 + 1 + 1$$

$$1 + 1 + 1 + 1 + 1 + 1$$

$$A. 1 + 1 + 1 + 1 + 1 + 1 B.$$

$$1 + 1 + 1 + 1 + 1 + 1$$

$$1 + 1 + 1 + 1 + 1 + 1$$

$$1 + 1 + 1 + 1 + 1 + 1$$

$$1 + 1 + 1 + 1 + 1 + 1$$

$$C. 1 + 1 + 1 + 1 + 1 + 1 D.$$

$$1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1$$

FROM the Process it is evident, that adding the several Columns, their Sums make 1, 2, 3, 4, 5, 6. which is the greatest, *viz.* *BC* (from which they descend in the same Order to *D*) and this *BC* is always the Number of Units in each Factor; so that if the Number of Units, multiplied into it self, had been 1000, then *BC* the last Term of the Progression ascending from *A* would have been 1000; and consequently if the Number of Units, multiplied into it self, had been  $\infty \times 1$ , then *BC* the last Term of the Progression would have been  $\infty \times 1$ . Now the Progression descends from *BC* to *D* in the same Order as it ascends from *A* to *BC*; but *ABC* is the Sum of the Progression 1, 2, 3, 4, 5, 6, &c. ending at *BC* =  $\infty \times 1$ ; and this Sum *ABC* is but half the Product (of which *BCD* is the other) that is to say, half the Square of Infinite: Thus you see that the Process

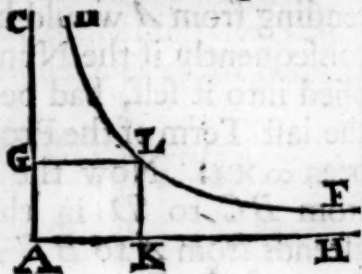
of *Art. 2.* of Case 1. agrees exactly with that of *Prop. 1.* of § 1. only that of *Art. 2.* Case 1. gives you the Progression twice over, and so makes it double of what it is in *Ex. 1.* *Prop. 1.* § 1.

*Schol.* From this Solution it appears that the Sum of the Progression in *Art. 3.* Case 1. viz.  $1+2+3+6+10+21$ , &c. is not precisely  $\infty^3$  or the Cube of Infinite for  $\frac{1}{1-1} \times$

$\frac{1}{1-1} \times \frac{1}{1-1} = 1+1+1+1$  &c.  $\times 1+1+1+1$  &c.  $\times 1+1+1+1$  &c. contains that Progression oftner than once; and the same is to be considered in all the Progressions, except when  $e=1$ .

§ VI. The Arithmetick of Infinites applied to Quadratures of curvilinear Spaces.

LET *DLF* be any of the Hyperbola's, whose Asymptots are *AC, AH*, let *AK=x*, *KL=y*, and the Equation comprehending all the Hyperbola's  $y x^n = 1$ .



By

By the common  
Methods,  $\left\{ \begin{array}{l} CAKLD = \frac{1}{1-n} \times x^{1-n} \\ HAGLF = \frac{n}{n-1} \times x^{1-n} \end{array} \right\}$

And also

*Corol.* 1. Let  $x = 1$ , then  $CAKLD = \frac{1}{1-n}$ ,

and  $HAGLF = \frac{n}{n-1}$

*Case* 1. Let  $n=1$ , so the Equation will be  $yx = 1$  for the common Hyperbola, in which  $CAKLD$ .

$$= \frac{1}{1-1} = 1 + 1 + 1, \text{ \&c. } = \infty \times 1.$$

And likewise  $HAGLF = \frac{1}{1-1} = 1 + 1 + 1, \text{ \&c. } = \infty \times 1$ . From whence it appears, that the Area of the *Apollonian* Hyperbola is infinite both ways.

*Case* 2. Let  $n=2$ ; so  $y x^2 = 1$  defines the next Hyperbola in which  $CAKLD = \frac{1}{1-2} = 1 + 2 + 4 + 8 + 16, \text{ \&c. } = \infty \times 2$ .

*Case* general. If you resolve  $\frac{1}{1-n}$  into a Series,

you will have  $CAKLD = \frac{1}{1-n} = 1 + n \times n^2$

$+ n^3 + n^4 + n^5, \text{ \&c. }$  and because the Terms of this Series are in a geometrical Progression ascending (supposing  $n < 1$ ) in the Ratio of 1 to  $n$ , there-

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$n$ , therefore by *Prop.* 2. §. 2. the Sum thereof must be  $1 + n + n^2 + n^3 + n^4 + n^5$ , &c.  

$$= \infty \times \frac{n}{n-1}$$

Now because  $n < 1$ , therefore  $\frac{n}{n-1}$  is  $< 1$ ; therefore in all these Hyperbola's (in which  $n < 1$ ) the Area *CAKLD* will be an infinite Number of equal Parts, each of which (*viz.*  $\frac{n}{n-1}$ ) is greater than 1. And hence is understood the meaning of the Geometers, who call these Spaces greater than infinite, that is, greater than  $\infty \times 1$  or infinite in general.

*Corol.* 2. In all the Hyperbola's (except the *Apollonian*) the infinite Area *CAKLD* (adjacent to the Asymptote *AC*) is equal to an infinite Number of the finite Area *HAGLF* (adjacent to the Asymptote *AH*) in the same Hyperbola.

*Demonstration.*  $CAKLD = \infty \times \frac{n}{n-1}$  by the general Case, but  $HAGLF = \frac{n}{n-1}$  by *Corol.* 1. § 6. Therefore

$$\{CAKLD = \infty \times HAGLF\} \quad Q. E. D.$$

*Problem.* Let  $\infty \times e$  be the infinite Area *CAKLD* of any given Hyperbola, it is required to find another Hyperbola, whose infinite Space

Space shall be to the Space ( $\infty \times e$ ) of the given Hyperbola in any given Ratio, as of  $p$  to 1. *Sol.* Let  $y x^n = 1$ . be the Equation of the Hyperbola sought, then by the general Case foregoing

its Area is  $= \infty \times \frac{n}{n-1}$ ; therefore by the Con-

dition of the Problem,  $\frac{n}{n-1} \cdot e :: p \cdot 1$ ; which

will give  $n = \frac{p e}{p e - 1}$ ; so that  $y x^{\frac{p e}{p e - 1}} = 1$

is the Equation to the Hyperbola sought; but  $p$  and  $e$  are given Numbers, and therefore this is a known Equation, and consequently the Hyperbola defined by it is also known

*Examp.* To find an Hyperbola whose Area shall be to that of the Apollonian, as 3 to 1. Now the Apollonian is  $= \infty \times 1$ , so  $e = 1$ , and  $p = 3$ , *Ergo* the Equation is  $y x^{\frac{3}{2}} = 1$ , whose Area by the general Case foregoing is  $= \infty \times 3$ , which is triple of the Apollonian.

*Prob. 2* To find an Hyperbola, whose interminated Space  $HKLF$  shall be equal to any given Number ( $a$ ) let  $y x^n = 1$  define the Hyperbola sought, where  $n < 1$ , then  $HAGLF =$

$\frac{n}{n-1}$  putting  $y = 1$  ( $= GL = GA$ .) by *Corol.*

of § 6. and because  $AGLK = 1$ , therefore it will be  $HAGLF - AGLK (= HKLE) =$

$\frac{n}{n-1} - 1$ . Therefore by the Condition of the

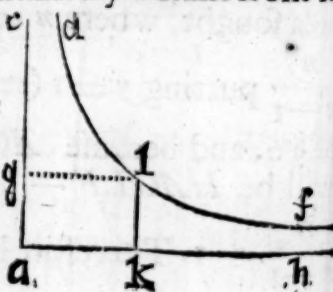
Problem



Problem  $\frac{n}{n-1} - 1 = a$ . which gives  $n = \frac{a+1}{a}$   
 for the Equation to the Hyperbola sought is  
 $\frac{y x^{\frac{a+1}{a}}}{y x^{\frac{a}{a}}} = 1$ .

Let (as before)  $AC, CH$  be the Asymptotes of any Hyperbola  $DLF$  defined by this Equation  $y x^n = 1$ , in which the Abscissa  $AK = x$ , and Ordinate  $KL = y$ , and  $n$  is suppos'd either equal to, or greater than Unity. 1° It appears that in all Hyperbola's the interminate Space  $CAKLD$  is infinite, and the interminate Space  $HAGLFL$  (except in the *Apollonian* where  $n = 1$ ) is finite. 2°. In every Hyperbola, one part of it continually approaches nearer and nearer to the Asymptote  $AC$ , and the other part continually nearer to the other Asymptote  $AH$ ; that is,  $LD$  meets with  $AC$  at a Point infinitely distant from  $A$ , and  $LF$  meets with  $AH$  at a Point infinitely distant from  $A$ .

3°. In two different Hyperbola's  $DLF, dl f$ , if we suppose  $n$  to be greater in the Equation of  $dl f$  than it is in the Equation of  $DLF$ , then  $LD$  shall meet sooner with  $AC$  than



$ld$  with  $ac$ , but  $Lf$  shall be longer in meeting with  $AH$  than  $lf$  with  $ab$ . Therefore since these Meetings are at infinite Distance from  $A$  and  $a$ , it follows necessarily that these infinite Distances must be one greater than another, viz.  $ac < AC$ , and  $ab > AH$ ; and in the same Hyperbola  $AC < AH$  and  $ac < ab$ , except the Apollonian in which  $AC = AH$ .

Therefore it must needs contribute very much to the right understanding the Scope of the foregoing Quadratures of these interminate hyperbolick Spaces, if we can determine the Length between the Centre  $A$  or  $a$  and the Point of Concourse of either Part of the Hyperbola, with that Asymptote to which it approaches.

*Problem 2.* To find the Point  $C$  where  $LD$  meets with the Asymptote  $AC$ , and the Point  $H$  where  $LF$  meets with the Asymptote  $AH$ , for any Hyperbola  $DLF$ , whose Equation  $y x^n = 1$  is given.

*Solution.* It is evident that the Ordinates  $LK$  ( $= y = \frac{1}{x^n}$ ) increase as the Abscissa's  $AK$  ( $= x$ ) decrease, so the last Ordinate must co-incide with the Asymptote  $AC$ , in which Case the Abscissa  $x = 0 = 1 - 1$ . Therefore  $AC =$

$$\frac{1}{1-1} = \frac{1}{x^n} \quad \text{Q. E. I.}$$

*Examp. 1.* To find  $AC$  in the common Hyperbola

perbola  $y = \frac{1}{x}$ . Because in this  $n=1$ , therefore

$$AC = \frac{1}{1-1} = (\text{by Art. 1. of } \S 5) 1+1+1+1,$$

$$\&c. = \infty \times 1.$$

*Examp. 2.* To find  $AC$  in the Hyperbola whose Equation is  $y = \frac{1}{x^2}$ . Because in this  $n$

$= 2$ , therefore from the general Solution  $AC$

$$= \frac{1}{1-1} \Big| 2 = 1+2+3+4+5+6, \&c. =$$

$$\frac{\infty^2}{2}; \text{ That is, } AC \text{ is an infinite Number of}$$

equal Parts, each of which is  $\infty \frac{1}{2}$ : So that  $AC$  in this is an infinite Number of equal Parts, each of which is  $\frac{1}{2} AC$  in the common Hyperbola.

*Schol.* If we could give the precise Sum of the Series in *Art. 3, 4, &c.* of *Case 1.* of  $\S 5$ , we should then have  $AC$  for all the other Hyperbola's; but that is not easily to be done, as is declared in the Scholium at the End of  $\S 5$ . Only this much we see (by *Case 4.* of  $\S 5$ .) That  $AC$  increases as the Powers of  $\infty$  whose Exponents are  $n$ . So in the Hyperbola

$y = \frac{1}{x}$ ,  $AC$  is as  $\infty$ ; in the Hyperbola  $y =$

$\frac{1}{x^2}$ ,  $AC$  is as  $\infty^2$  in the Hyperbola  $y = \frac{1}{x^3}$ ,  $A$

$C$  is  $\infty^3$ ; and so on.

*Part.*

*Part. 2d.* To find  $AH$  when  $LF$  meets with its Asymptote  $AH$ . Here we must consider  $GL (=x)$  as the Ordinate, and  $AG (=y)$  as the Abscissa. Now when  $GL$  becomes  $AH$ , then  $y=0$ . But univerfally  $GL (=x)$

$$= \frac{1}{y^{\frac{1}{2}}} \text{ therefore } AH = \frac{1}{0^{\frac{1}{2}}} = \frac{1}{1-1|^{\frac{1}{2}}}$$

put  $\frac{1}{n} = e$ , and then by *Lem.* of § 5.  $AH =$

$$\frac{1}{1-1} e = 1 + e + \frac{e \times e + 1}{2} + \frac{e \times e + 1 \times e + 2}{2 \times 3} + \frac{e \times e + 1 \times e + 2 \times e + 3}{2 \times 3 \times 4} \text{ \&c.}$$

But ſince we cannot aſſign the Sum of theſe Progreſſions, therefore this Series is of no Uſe in the Solution of the Problem. We ſhall therefore conſider the Problem (as to the finding  $AH$ ) under two Caſes, firſt when  $n$  is an Integer, and ſecondly when it is a Fraction.

*Caſe 1.* When  $n$  is an Integer. So if  $n=1$  (as

in the common Hyperbola) then  $AH (= \frac{1}{1-1})$

$= 1 + 1 + 1, \text{ \&c. } = \infty \times 1 = AC.$  If  $n=2$ ,

then  $AH (= \frac{1}{1-1}^{\frac{1}{2}} = \sqrt{1-1}) = \sqrt{\infty \times 1}$  is

a mean proportional between 1 and infinite.

If  $n=3$ , then  $AH (= \frac{1}{1-1}^{\frac{1}{3}} = \sqrt[3]{1-1}) = \sqrt[3]{\infty}$

$\sqrt[n]{\infty \times 1}$  is the first of two mean Proportionals between 1 and Infinite. And universally if between 1 and Infinite there be supposed as many mean Proportionals as there are Units in  $n-1$ ,

then  $AH (= \frac{1}{1-1} \frac{1}{n} = \sqrt[n]{\frac{1}{1-1}} = \sqrt[n]{\infty \times 1})$

shall be the first of these Means.

*Case 2d.* When  $n$  is a Fraction, suppose  $n = \frac{p}{q}$ , but  $p < q$ , because we always suppose  $n < 1$  except in the *Apollonian* where  $n = 1$ . So then

$\frac{1}{n} = \frac{q}{p}$ , therefore  $AH = \frac{1}{1-1} \frac{1}{\frac{q}{p}} = \sqrt[p]{\frac{1}{1-1} \frac{q}{p}} =$

$\sqrt[p]{\infty \times 1} \frac{q}{p} = \sqrt[p]{\infty \times 1} \frac{q}{p}$ . So for *Ex.* if  $n = \frac{3}{2}$ , viz.

$p=3, q=2$ , then  $AH = \sqrt[3]{\infty \times 1} \frac{2}{3}$

*Corol.* From both these Cases it is evident, that the greater Number we suppose  $n$  to be, so much the sooner will  $LF$  meet with  $AH$ : for the greater we suppose  $n$  to be, so much the less will  $\frac{1}{n}$  or  $(\frac{q}{p})$  be; but  $\frac{1}{n}$  (or  $\frac{q}{p}$ ) is the Exponent of  $\frac{1}{1-1}$  (or  $\infty$ ) which gives the Value of  $AH$ . *Ergo* the greater that  $n$  is, the shorter will  $AH$  be. *Q. E. D.*



*Schol.* Since in all Hyperbola's  $AH = \frac{1}{1-1^n}$

$\frac{1}{1-1} = \frac{p}{1-1} = \sqrt{1-1}^q = \sqrt{\infty \times 1}^q$  is some mean Proportional between 1 and  $\infty \times 1$ , it is worth the while to consider what sort of Numbers these means are, *viz.* whether they be finite, infinite, or neither. It is certain they cannot be finite, for then it would follow that the square of a finite Number should be infinite which is absurd. Nor can they be properly esteemed infinite, for the equal part of such an Infinite must be 1 divided by such a mean.

*Demon.* Let  $a$  denote that equal part, then

$\infty \times a = \sqrt[p]{\infty 1^q}$ , which give  $a = \frac{1}{\sqrt[p]{\infty}^{p-q}}$

*Q.E.D. (Ex.gra.)*  $\sqrt{\infty \times 1}$  or a mean between

and infinite must be  $\infty \times \frac{1}{\sqrt{\infty}}$  or an infinite

Number of equal Parts, each of which is  $\frac{1}{\sqrt{\infty}}$

or 1 divided by the mean between 1 and  $\infty \times 1$

so that this equal part  $\frac{1}{\sqrt{\infty \times 1}}$  is neither finite

nor infinitely small, nor infinitely great, and

consequently the mean  $\sqrt{\infty \times 1}$  is not properly an infinite Number. And hence we see that

PART II.

M

there

there are Progressions of Numbers whose Sums are neither finite nor infinite, but between both.

*Ex. gra.*  $\sqrt{\infty} \times 1$  where  $p=2, q=1$ , so  $\sqrt{\infty} \times 1$

$\frac{1}{1-1} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots$  &c. is a Num-

ber neither finite nor infinite: And so all the means between 1 and Infinite, are neither finite nor infinite.

I have hitherto consider'd only these Cases in which  $n =$  or  $<$  1; let us now see what would be the Result if we suppose  $n = 0$ . It is evident then that the general Equation, viz.  $yx^n = 1$  to all Hyperbola's would in this Case be  $y = 1$ , so that the Hyperbola's will in this Case degenerate into a strait Line  $DLF$ , parallel to  $AH$ . But considering it as the simplest Hyperbola, let us consider at what distance it will meet with its



Asymptotes. Now universally  $AC = \frac{1}{n}$

the Sol of part. 1. of *Prob. 3.*) when  $x = c$ , but  $n = 0$  by Supposition: *Ergo*, be  $x$  what it will  $x_0 = 1$ , *Ergo*,  $AC = 1$ , 2°. From the Solution of part 2. of *Prob. 3.* we have found that

$AH = \frac{1}{1-1} \frac{1}{n}$  which in this Case will give

$AH = \frac{1}{1-1} \frac{1}{1} = \frac{1}{1-1}$ , that is  $AH = \infty$

or the infinite Power of infinite.

*Corol. 1.* Considering a strait Line as an Hyperbola, it can have but one Asymptote, *viz.*  $AH$ , whose Concourse with  $AH$  is at greater Distance from  $A$  than any other Hyperbola whatsoever.

*Corol. 2.* All the Hyperbola's  $y = \frac{1}{x-n}$  whose Asymptotes are  $AC, AH$  must intersect  $DF$ , and the greater  $n$  is, so much the more they bend towards  $AH$  and recede from  $DF$ ; and as they all intersect  $DF$ , so they all intersect one another in one Point only; and after the Intersection, that, in which  $n$  is greatest, still falls lowest or approaches nearest to  $AH$ .

*Schol.* If you suppose  $n$  infinitely great, *viz.*  $y = \frac{1}{1-n}$ , then the Hyperbola will be a strait Line parallel to the Asymptote  $AC$ .

*Sect. 7.* We have now finished this Business of the Hyperbola's which has afforded us a new Speculation of Numbers, *viz.* of such as are neither finite nor infinite, which deserves to be consider'd better than either my Time or my Capacity will permit. However, I shall here set down a few thoughts about them, till I have more leisure to prosecute them.

*First* then, to distinguish them from finite and infinite, I shall call them *indefinite* Numbers, and denote them by this Sign  $\propto$ .

2<sup>o</sup>. *Indefinite* Numbers I suppose to be intermediate Numbers lying between finite and infinite: For as we do not descend from 1 to 0

at one Step, but must pass through an infinite Series of Fractions,  $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \&c.$  So it is impossible that in ascending from 1 to  $\infty$ , we should pass immediately from finite to infinite; therefore the Steps between these two are indefinite Numbers; thus before we arrive from 1 to  $\infty$  1, we must come at  $\sqrt{\infty}$  1, and before we come at  $\sqrt{\infty} \times 1$  we must first come at  $\sqrt[3]{\infty} \times 1$  and so on.

3°. The Rules for the Arithmetick of Indefinites may be made after the same manner as is done for that of infinites by D. Chyne. To which I shall add, that an Indefinite as  $\infty \frac{q}{p}$  or

$\sqrt[p]{\infty} \frac{q}{p}$  multiplied by another Indefinite as  $\infty \frac{r}{s}$  makes the Product  $\infty \frac{q}{p} + \frac{r}{s}$  this Product be-

comes infinite when it happens that  $qs + pr < \text{or} = ps$ , but the Product is only indefinite when  $qs + pr > ps$ . And if an indefinite as  $\infty \frac{q}{p}$  be divided by another inde-

finite as  $\infty \frac{r}{s}$ , the Quotient  $\infty \frac{p}{q} - \frac{r}{s}$  is

infinite when  $qs - pr$  is  $< \text{or} = ps$ ; but it is finite when  $qs - pr = 0$ , and indefinite when  $qs - pr > ps$ .

Note, THAT in expressing an indefinite Number by  $\infty \frac{p}{q}$ , I always suppose the

Nume

Numerator  $q$  less than the Denominator  $p$ ; for if  $q$  be either  $<$  or  $= p$ , then  $\infty | \frac{p}{q}$ , is an Infinite. That these Numbers are not infinite, may be thus demonstrated. If (*ex.*

*gr.*)  $\sqrt{\infty \times 1}$  were infinite, then  $\infty \times \frac{1}{\sqrt{\infty}}$

( $= \sqrt{\infty \times 1}$ ) must be infinite, and also  $\frac{1}{\sqrt{\infty}}$

must be  $= 0$ ; *Ergo*  $\infty \times 0 = \infty \times \frac{1}{\sqrt{\infty}}$

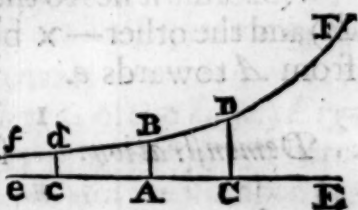
( $= \sqrt{\infty}$ ) = infinite, but  $\infty \times 0 = 1$ ; *Ergo*  $1 =$  infinite, which is absurd.

*Seet.* 8. Containing some miscellaneous Things relating to infinites.

*Lem.* Let  $FBDde$  be a Logarithmick Curve whose first Ordinate  $AB = 1$ , Ab-*scissa*  $AC = x$ , Ordinate  $CD = y$ , and Asymptote  $Ee$ : Now from the known Property of this Curve, it follows,

1°. THAT  $AC$  are the Logarithms of  $CD$ , *i. e.* If you make the Ordinate  $y = CD$  represent a Number, then its Abscissa  $x (= AC)$  shall be the Logarithm thereof.

2°. THAT the Logarithm of 1 is nothing; for





the first Ordinate  $AB = 1$ , but its Abscissa is  $= 0$ , therefore (by 1°) the Log. of  $1 = 0$ .

3°. THAT the Ordinates  $cd$  (to the left of  $AB$ ) denoting Fractions, their respective Abscissa's  $Ar$  are the Logarithms of these Fractions: So that as  $x$  is the Log. of any Integer,  $CD$  in the like manner  $-x$  denotes the Log. of any Fraction ( $c d$ ).

4°. THAT the Log. of any Fraction  $\frac{1}{n}$  is e-

qual to  $-\text{Log. of } n$ . So that the Log. of any Fraction (whose Numerator is 1) is equal in Magnitude to the Log. of that whole Number, which is the Denominator; there being no Difference between the Log. of that Fraction  $\frac{1}{n}$

and the Log. of this Integer  $n$ , but that this is  $+x$  (because it lies to the right from  $A$  towards  $E$ ) and the other  $-x$  because it lies to the left from  $A$  towards  $e$ .

*Demonstration.*  $\frac{1}{n}$  signifies 1 divided by  $n$ , therefore by the Rules of Division  $l: 1 - l: n = \text{Log. of the Quotient, viz. } l: \frac{1}{n}$ . But  $l: 1 = 0$  (by 2°) Ergo  $-l: n = l: \frac{1}{n}$ . Q. E. D.

5°. If this Curve be continued (*utrinque*) from  $B$  in *infinitum*, then  $f$  will meet with the Asymptote at an infinite Distance  $Ae$ : But it will

will diverge from the Asymptote on the other side, so that at an infinite Distance  $AE$  the last Ordinate  $EF$  will be infinite. And since the last Ordinate  $ef$  is  $= 0$ , whose Abscissa  $Ae = \infty$ , it is evident that the Log. of  $0$  is  $= \infty$  or rather  $= -\infty$ : It is evident likewise the last Ordinate  $EF$  is  $= \infty$ , whose Abscissa  $AE$  is also  $= \infty$ . So that the Log. of  $0$  and the Log. of  $\infty$  are equal, only the one is  $+\infty$  and the other  $-\infty$ .

*Prop.*  $1. \frac{1}{1-1} = \infty = 1 + 1 + 1 + 1 + 1, \text{ \&c.}$

This may seem absurd, but the Demonstration is evident from the foregoing *Lemma*. For  $\frac{1}{1-1}$

$= -\frac{1}{0}$ . Let then  $\frac{1}{1-1} = y$ . That is  $\frac{1}{1-1} = y$ .

This reduc'd to a logarithmical Equation gives  $-1 \times l: 1-1 = l: y$ ; that is  $-1 \times l: 0 = l: y$ . But  $l: 0 = -\infty$  (by *Art.* 5. of the *Lem.*) *Ergo*  $-1 \times -\infty = l: y$ , that is  $+\infty = l: y$ . Therefore  $y$  is infinite (*sc.*  $y = \infty$ ) for no Number has an affirmative and infinite Logarithm, except an infinite Number. Since then  $y = \infty$ , then

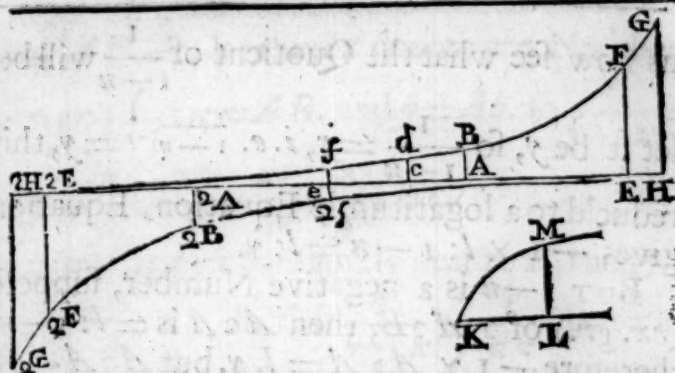
$\frac{1}{1-1} = \infty$  that is  $\frac{1}{0} = \infty$ . *Q. E. D.*

*Schol.* But by  $0$  cannot be understood absolute nothing, for an infinite Number of absolute Nothings cannot make  $1$ ; but by  $0$  is understood an infinitely small part, as in the *calc.*

*diff.*  $dx$  is an infinitely small part of  $x$ , so that  $dx$  is as 0 to  $x$ : Not that  $dx$  is absolutely nothing, for it is divisible into an infinite Number of Parts, each of which is  $d dx$ . And therefore the Demonstration, which supposing  $f$  and  $e$  meeting at an infinite Distance  $Ae$ , makes the last Ordinate  $ef = 0$ , implies no more but that  $ef = dx$ . But then it may be inquir'd what is the Quotient that arises from the Division of 1 by absolute Nothing. I say there is no Quotient because there is no Division: Therefore it is a Mistake to say the Quotient is 1 or Unity undivided, which is demonstrably false, neither is the Quotient  $= 0$ . For properly speaking there is no Quotient, and therefore it is an Error to assign any. In like manner, it is an Error to say, that  $0 \times a$  makes the Product 0; for properly speaking there is no Product. It is true, this of Multiplication has no influence upon Practice, but that of Division has.

FROM hence it appears, that a Curve is said to meet with its Asymptote, when the Ordinate is infinitely little.

THIS



THIS same Notion does explain how it comes to pass that 1 divided by a negative Number gives a Quotient greater than infinite. For if (when you call  $LM=x$ ,  $KL=y$ .)  $fe=dx$ , then because the Points  $f$  and  $e$  are infinitely near, we may conceive the Logarithmick Curve continued as intersecting  $AH$  in the Point  $e$ ; so that  $FBf_2f_2B_2F_2G$  makes but one continued Curve, whereof the Part above represents the affirmative Numbers by its Ordinates, and the negative Numbers are represented by the Ordinates of the Part below; but  $A$  is the Beginning of the Axis for the Logarithms of both, *viz.*  $AE$ ,  $AH$ , &c. are the Logarithms of the affirmative Integers  $FE$ ,  $GH$ , &c.  $Ae$  the Logarithm of any affirmative Fraction,  $fe$  the Logarithm of an infinitely little Fraction,  $fe=dx$ .  $A_2A$ ,  $A_2E$ , &c. The Logarithms of the negative Numbers,  $2A_2B$ ,  $2E_2F$ , &c.

Now let  $n$  be any Number greater than Unity, then  $1-n$  will be a negative Number, let us

us now see what the Quotient of  $\frac{1}{1-n}$  will be,

let it be  $y$ , so  $\frac{1}{1-n} = y$ , i. e.  $1-n = \frac{1}{y}$ , this

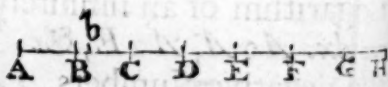
reduc'd to a logarithmic Equation, Equation gives  $-1 \times l: 1-n = l: y$ .

BUT  $1-n$  is a negative Number, suppose (ex. gr.) of 2  $A \geq B$ , then  $A \geq A$  is  $= l: 1-n$ , therefore  $-1 \times A \geq A = l: y$ . but  $A \geq A < A$ , and  $Ae = -\infty$ , therefore  $A \geq A < -\infty$ . Let then  $A \geq A = -\infty \times a$  (where  $a < 1$ ) then  $-1 \times -\infty a = l: y$ , that is  $\infty \times a = l: y$ , but  $\infty \times a < \infty$ , Ergo  $y$  is a Number greater than infinite.

And here it is observable, that there are affirmative Numbers less than nothing, denoted by the several Powers of  $dx$ , as  $dx^2$ ,  $dx^3$ , &c. or by the second, third, &c. Differences, and these Numbers may be aptly represented by the Ordinates of the logarithmic Curve, continued from  $f$  towards  $H$  when  $dx^n$  is affirmative, or from  $f$  towards  $H$  when  $dx^n$  is negative.

Another way of explaining what is meant by  $\frac{1}{0} = \infty$ .

Let  $AH$  produced indefinitely be divided into equal Parts  $AB, BC, CD, DE$ , &c. so that a Part of this Line shall denote any Number, suppo-





sing  $AB=1$ , let then  $x$  denote any Number,  
(*ex. gr.*) Let  $x=AB$ . and  $y=Ab$ . so  $\frac{1}{y-x}=$

$$\frac{1}{y} + \frac{x}{y^2} + \frac{x^2}{y^3} + \frac{x^3}{y^4} + \frac{x^4}{y^5}, \text{ \&c.}$$

Now suppose  $b$  infinitely near to  $B$ , then  $y-x$   
 $= Bb = dx$ , so that  $\frac{1}{y-x} = \frac{1}{dx}$ . But  $x =$   
 $dx + dx + dx$ , \&c. That is  $1 \times x = dx \times 1 + 1$   
 $+ 1 + 1$ , \&c. And therefore  $\frac{1}{dx} = \frac{1}{x} + \frac{1}{x} +$   
 $\frac{1}{x} + \frac{1}{x}$  \&c. But by Supposition  $x = AB$   
 $= 1$ . Ergo  $\frac{1}{dx} = 1 + 1 + 1 + 1$ , \&c.  $= \infty \times 1$ .

But  $\frac{1}{0} = 1 + 1 + 1 + 1$ , \&c. Ergo  $\frac{1}{0} = \frac{1}{dx}$ , or  
 $dx = 0$ , but  $dx$  is not absolute nothing, and  
therefore when we say  $\frac{1}{0} = 1 + 1 + 1$ , \&c.

$0$  does not denote absolute nothing, but only  
 $dx$  or an infinitely small part of  $x$ . And there-  
fore when (in the Quotient  $\frac{1}{y-x}$ ) we say let  
 $y=x$ , the meaning is not that  $y$  is absolutely  
equal to  $x$ , (for then there would be no Divi-  
sion, and consequently no Quotient) but only  
that  $y$  exceeds  $x$  by an infinitely small Quantity  
 $dx$ , which is sufficient to make them equal.  
I have

I have hitherto considered no Progressions of Numbers except these that are in arithmetical and geometrical Progressions, and these

that arise from  $\frac{1}{1-1} e$  as in § 5. I shall now

proceed to treat of other Progressions, and these, which offer themselves first, are the several Powers of Numbers in an arithmetical Progression. Let then  $z$  denote the Sum of any arithmetical Progression  $a, a+x, a+2x, a+3x, a+4x, \&c.$   $A$  the Sum of their Squares,  $B$  the Sum of their Cubes,  $C$  the Sum of their Biquadrates,  $\&c.$  that is, let

$$z = a + (a+x) + (a+2x) + (a+3x) + (a+4x) \&c.$$

$$A = a^2 + \overline{a+x}^2 + \overline{a+2x}^2 + \overline{a+3x}^2 + \overline{a+4x}^2 \&c.$$

$$B = a^3 + \overline{a+x}^3 + \overline{a+2x}^3 + \overline{a+3x}^3 + \overline{a+4x}^3 \&c.$$

$$C = a^4 + \overline{a+x}^4 + \overline{a+2x}^4 + \overline{a+3x}^4 + \overline{a+4x}^4 \&c.$$

And so on to higher Powers. Now in order to find the Sum of any of these Progressions ascending till the last Term is  $= \infty \times 1$ , I shall premise these following *Lemma's*, by the Help whereof you may find  $z, A, B, C, \&c.$  for any Number of Terms. Calling therefore  $t$  the Number of Terms in each Series, and  $v$  the last Term of the first whose Sum is  $z$ . I say

$$\text{Lem. 1. } z = \frac{v^2 \times xv + ax - a^2}{2x} \quad \text{Lem.}$$

$$\text{Lem. 2. } A = \frac{v+x|^3 - a^3 - tx^3 - 3x^2 z}{3x}$$

$$\text{Lem. 3. } B = \frac{v+x|^4 - a^4 - tx^4 - 4x^3 z - 6x^2 A}{4x}$$

$$\text{Lem. 4. } C = \frac{v+x|^5 - a^5 - tx^5 - 5x^4 z - 10x^3 A - 10x^2 B}{5x}$$

In each of which  $tx = v + x - a$  (by *Lem. 1.* § 1.) and it is easy to continue these *Lemma's* for higher Powers.

Before we apply these *Lemma's* to the finding the Sum of any Progression whose last Term is infinite, it will be necessary to substitute the Values of  $t, z, A, B, C, \&c.$  When you have made this Substitution, you must reject out of the *Lemma's* every Term in which  $v$  does not occur; for the last Term being infinite will make  $v$  an indefinite Number, and since all the other Terms (in which  $v$  is not) are finite, therefore they are as nothing in Respect of  $v$ , and consequently to be rejected. So, for Progressions whose last Term is  $= \infty$ , the *Lemma's* will be.

$$\text{Lem. 1. } z = \frac{v^2 + xv}{2x}$$

$$\text{Lem. 2. } A = \frac{v^3 - x^2 v}{3x} - \frac{v^2 - xv}{2}$$

*Lem.*

$$\text{Lem. 3. } B = \frac{v^4 - 2xv^3 + x^2v^2 + 2x^3v}{v^4 - 2xv^3 + x^2v^2 + 2x^3v}$$

$$\text{Lem. 4. } C = \frac{v^5}{5x} + \frac{15v^4 + 10xv^3 - 59x^3v}{30}$$

LET us now consider what the Value of  $v$  is, in each of the *Lemma's*  $A, B, C$ , &c. An Example or two will make the Thing plain.

*Ex. 1.* Let  $\{1, 2, 3, 4, 5, \&c. \text{ to } v,$   
whose Sum is  $z$ .

Then for *Lem. 2.*  $\{1, 4, 9, 16, 25, \&c. \text{ to } v,$   
whose Sum is  $A$ .

BUT  $v^2 = \infty$  (by *Supp.*) Therefore  $v = \sqrt{\infty} = \infty^{\frac{1}{2}}$  so that  $\infty^{\frac{1}{2}}$  is the Value of  $v$  in *Lem.*

*2d.* In like manner  $\sqrt[3]{\infty}$  or  $\infty^{\frac{1}{3}}$  is the Value of  $v$  in *Lem. 3.* And universally if  $e$  denote the Exponent of the Power to which each of the Terms of any arithmetical Progression

is raised, then  $v = \sqrt[e]{\infty} = \infty^{\frac{1}{e}}$ . in the *Lemma* that gives the Sum of these Terms (whose last is  $\infty$ ) raised to this Power.

*Prop. 1.* To find the Sum of a Series of Numbers (continued till the last Term is  $\infty$ ) whose Terms are the Squares of any arithmetical Progression.

By *Lem. 2.* the Sum sought is  $A = \frac{v^3 - x^2v}{3x}$

$$3x$$

$v^2 - xv$ , but in this Case  $v = \sqrt{\infty}$  or  $\infty^{\frac{1}{2}}$

therefore all the Terms except the highest must  
evanish, and consequently the Sum of the

$$\text{Squares } A = \frac{v^3}{3x} = \frac{\infty^{\frac{1}{2}|3}}{3x} = \frac{\infty^{\frac{3}{2}}}{3x} = \frac{\infty^{\frac{1}{2}}}{x}$$

*Examp.* Let  $x$  be  $= 1$ , so  $1, 2, 3, 4, \&c.$  to  $\sqrt{\infty}$  is the arithmetical Progression, the Sum ( $A$ ) of whose Squares is sought; I say  $1 + 4 + 9 + 16 \&c. \infty \times \frac{1}{3} \sqrt{\infty}$ , that is, the Sum of the Progression is an infinite Number of equal Parts, each of which is  $\frac{1}{3} \sqrt{\infty}$ .

*Prop. 2.* To find the Sum of the Cubes of any arithmetical Progression.

In this Case  $v = \sqrt{\infty}$  or  $\infty^{\frac{1}{3}}$ . Therefore in  
*Lem. 3.* all the Terms except the highest must

$$\text{evanish, so that } B = \frac{v^4}{4x} = \frac{\infty^{\frac{4}{3}}}{4x} = \frac{\infty^{\frac{1}{3}}}{x}$$

*Ex.* Let  $x = 1$ , so  $1, 2, 3, 4, 5, \&c.$  to  $\sqrt{\infty}$  is the arithmetical Progression, the Sum ( $B$ ) of whose Cubes is sought, I say  $1 + 8 + 27 + 64$

$$+ 125 \&c. = \infty \times \frac{\infty^{\frac{1}{3}}}{3}$$

*Prop.*



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*Prop. 3.* To find the Sum of the Biquadrates of any arithmetical Progression.

IN this Case  $v = \sqrt[4]{\infty}$  or  $\infty^{\frac{1}{4}}$ . Therefore by

$$\text{Lemma 3d. } C = \frac{v^5}{5x} = \frac{1}{5x} \infty^{\frac{5}{4}} = \frac{1}{5x} \infty^{\frac{5}{4}} \\ = \infty \times \frac{\infty^{\frac{1}{4}}}{5x}$$

*Ex.* Let  $x=1$ , so 1, 2, 3, 4, &c. to  $\sqrt[4]{\infty}$  is the arithmetical Progression; I say then that  
 $1+16+81+256$  &c.  $= \infty \times \frac{\infty^{\frac{1}{4}}}{5}$ .

*Prop. 4. Universal.* Let  $e$  denote the Exponent of any Power, to which the Terms of any arithmetical Progression  $a, a+x, a+2x, a+3x$ , &c. are raised, and it is required to find the Sum of the Terms so raised. I say  
 $a^e + a+x|^e + a+2x|^e + a+3x|^e + a+4x|^e$  &c.  
 $= \infty \times \frac{\infty^{\frac{1}{e}}}{e+1 \times x}$ .

*Scholium.* From hence may be easily deduc'd the Quadratures of all the Parabola's  $z=y^e$  (where  $z$  is the Ordinate,  $y$  the Abscissa, and  $e$  an affirmative Number) which is one remarkable Use of this Arithmetick of  
Infi-

Infinites. For if in this Figure  $AB = y$ ,  $BC = z$ , and  $z = y^e$ , it is plain that if you put  $y = 1, 2, 3, 4, 5, \&c.$  successively, then will  $z = 1^e, 2^e, 3^e, 4^e, 5^e, \&c.$  successively; which shews that the Ordinates are the Terms of an arithmetical Progression rais'd to a Power, whose Exponent is  $e$ , and that  $(x)$  the common Difference is 1: But if the first Abscissa  $y = 1$  be infinitely (or indefinitely small) then the Ordinates will be infinitely near to one another, and the last Ordinate  $BC$  will be an  $\infty$  Number of these Ones. Therefore since to find the Area  $ABC$ , is, in effect, to find the Sum of the Ordinates, and since the Ordinates are  $1^e, 2^e, 3^e, 4^e, 5^e, \&c.$  till you come to the last, which is  $BC = \infty$ . Therefore by Prop.



$$4. ABC = \infty \times \frac{\infty^{\frac{1}{e}}}{e+1}; \text{ that is } ABC = BC \times$$

$\frac{BC^{\frac{1}{e}}}{e+1}$ , the same which is found by the ordinary Methods of Quadratures. And note that the whole Abscissa  $AB$  denotes the Number of the Terms.

N. B. The Quadrature of all Sorts of Curves express'd by one Term thus assign'd, it is easy by the Method of Assumptions of Series's to extend this Method to all Sorts of Quadratures hitherto discovered.

## ADDITIONS.

I have shewn in *p.* 160. *Ec.* that mean Proportionals between 1 and  $\infty$  are neither finite nor infinite. For a farther Illustration of what I have said upon this Head, let us consider what these Progressions are, whose Sum is more than finite and less than infinite.

*Prop.* 1.  $\sqrt{\infty}$  is a Mean between 1 and infinite; but  $\infty = \frac{1}{1-1}$ , therefore  $\sqrt{\infty} =$

$$\frac{1}{\sqrt{1-1}} = \frac{1}{1-1}^{\frac{1}{2}} \text{ which by Sir Isaac Newton's}$$

$$\text{Theorem gives } \sqrt{\infty} = \frac{1}{1-1}^{\frac{1}{2}} = 1 + \frac{1}{2} + \frac{1 \times 3}{2 \times 4} + \frac{1 \times 3 \times 5}{2 \times 4 \times 6} + \frac{1 \times 3 \times 5 \times 7}{2 \times 4 \times 6 \times 8}, \text{ \textit{Ec.} that is}$$

$$\sqrt{1+1+1}, \text{ \textit{Ec.} } = 1 + \frac{1}{2} + \frac{1 \times 3}{2 \times 4} + \frac{1 \times 3 \times 5}{2 \times 4 \times 6} + \frac{1 \times 3 \times 5 \times 7}{2 \times 4 \times 6 \times 8}, \text{ \textit{Ec.}}$$

AND so (by squaring each Side of this Equation) you will find it in Fact to be  $1+1+1$ , *Ec.*  $= 1+1+1$ , *Ec.* From hence it appears that the Progression, whose Sum (being neither finite nor infinite) is  $\sqrt{\infty}$  consists of Fractions, whose Numerators are the Products of the continual Multiplication of

*Carol.* In like manner you may find  $\sqrt[\text{B}]{\infty}$   
 $\sqrt{\infty}$  &c. by resolving  $\frac{\text{I}}{\sqrt{\text{I}-\text{I}}} = \frac{\text{I}}{\text{I}-\text{I}} \sqrt{\frac{\text{I}}{\text{I}-\text{I}}}$

$$\sqrt{\infty} = \frac{1}{\sqrt{1-1}} = \frac{1}{0} = 1 + \frac{1}{3} + \frac{1 \times 4}{3 \times 6}$$
$$\sqrt{\infty} = \frac{1}{5} = 1 - 1 \quad 1 + \frac{1}{5} + \frac{1 \times 6}{5 \times 10}$$
$$+\frac{1 \times 6 \times 11}{5 \times 10 \times 15} \text{ Cr. } \sqrt{\infty} = \frac{1}{\sqrt{1-1}} = 1-1 \left| \frac{1}{0} \right.$$
$$\frac{1}{e} = 1 + \frac{1}{e} + \frac{1 \times e + 1}{e \times 2e} + \frac{1 \times e + 1 \times 2e + 1}{e \times 2e \times 2e}$$
$$+ \frac{1 \times e + 1 \times 2e + 1 \times 3e + 1}{e \times 2e \times 3e \times 4e} + \frac{1 \times e + 1 \times 2}{e \times 2e \times 3}$$
$$\frac{e+1 \times 3e+1 \times 4e+1}{\times 4e \times 5e} \text{ \&c.}$$

*Prop. 2.* Let 1 be the first,  $r$  the second, and  $v$  the last Term of a geometrical Progression 1,  $r$ ,  $r^2$ ,  $r^3$ ,  $r^4$ , &c. and  $t$  the Number of Terms, then  $v = r^{t-1} = \frac{r^t}{r}$ , which gives  $r^t = rv$ .

*Corol.* Let  $\tau$  be the Number of Terms, and  $v$  the last Term in the like Progression 1,  $s$ ,  $s^2$ ,  $s^3$ ,  $s^4$  &c. then  $v = s^{\tau-1}$ .

*Corol. 2.* From these two Values of  $v$ , I have this Equation  $r^{t-1} = s^{\tau-1}$ ; whence it appears that if  $s$  be greater than  $r$ , then will  $\tau < t$ , that is, in any two Progressions whose first Terms are the same, that, whose second Terms is greater than  $r$  the second Term of the other, will sooner arrive at any given Number  $v$ .

*Example.* Let  $r = 2$ , then 1, 2, 4, 8, 16. Let  $s = 4$ , then 1, 4, 16, that is, there are 5 Terms (or  $t = 5$ ) in arriving at 16 in the first Progression, whereas in the second ( $\tau = 3$ ) it arrives at 16 in the 3<sup>d</sup> Term.

*Problem.* Having  $r$ ,  $t$ ,  $s$ , to find  $\tau$  in which they shall both have the same last Term  $v$ .

*Sol.*  $s^{\tau-1} = r^{t-1}$  (by *Corol. 2.*) this turn'd into a logarithmical Equation is  $\tau - 1 \times l s =$

$$\frac{t - 1 \times l r}{l s} = \frac{t l r - l r}{l s} + 1. \text{ Q. E. I.}$$

*Corol.*



*Corol.* Because by *Prop.* 2.  $r = rv$ , therefore  $t\ lr = lrv$ , so substituting  $lrv$  in place of  $t\ lr$ , we shall have  $\tau = \frac{lrv - lr + ls}{ls} =$

$$\frac{lrv}{ls} - \frac{lr}{ls} + 1.$$

*Corol.* 2. Because by *Corol.* 1.  $t\ lr = lrv$ , therefore  $t = \frac{lrv}{lr}$ . And consequently

$$t - \tau = \frac{lrv}{lr} - \frac{lrv}{ls} + \frac{lr}{ls} - 1.$$

But by the known Property of Numbers we have  $lr + lv = lrv$ , therefore  $t - \tau = \frac{lr + lv}{lr} - \frac{lr + lv}{ls} + \frac{lr}{ls} - 1$  that is  $t - \tau =$

$$1 + \frac{lv}{lr} - \frac{lv}{ls} - 1, \text{ that is, } t - \tau = \frac{lv}{lr} - \frac{lv}{ls}.$$

*Corol.* 3. Let the last Term  $v$  be infinite or  $l: \infty$

$\infty$ , then  $t - \tau = \frac{lv}{lr} - \frac{lv}{ls}$ ; which shews

how much sooner the Progression 1,  $s$ ,  $s^2$ ,  $s^3$ ,  
N 3

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$s^3, \&c.$  will arrive at  $\infty$ , than  $1, r, r^2, r^3, \&c.$  that is, how many more Terms there is in  $1, r, r^2, r^3, \&c.$  than in  $1, s, s^2, s^3, \&c.$  when the last in both is  $\infty$ .

*Prop. 3.* Let  $BG = x$   $GEA$   
 $= y; y \propto x^s = 1$ , where  $s$  is a  
 ny Number greater than U-  
 nity. Then by the *Calculus*  
*Integralis*, we have  $DEG$



$BA = \frac{x^{1-s}}{1-s}$ , so putting  $x = 1$ . we have

$$DEGBA = \frac{1}{1-s}$$

$$\text{But } \frac{1}{1-s} = 1 + s + s^2 + s^3 + s^4 \&c. \\ = DEGBA.$$

But calling  $v$  the last Term of this ascend-  
 ing geometrical Progression  $1 + s + s^2 + s^3$   
 $+ \&c.$  we shall (as is shewn in Page 136.  
 of this Chapter) have  $1 + s + s^2 + s^3 +$

$$s^4 + \&c. = \frac{s^v}{s-1}, \text{ therefore } DEGBA =$$

$$\frac{s^v}{s-1}. \text{ So that we have three different Expres-}$$

$$\text{sions of the Area } DEGBA, \text{ viz. } \frac{1}{1-s},$$

$s^v$

$\frac{sv}{s-1}$ , and the Series  $1 + s + s^2 + s^3 + s^4 \&c.$   
 $s-1$

But what this Number  $v$  is (which denotes the last Term of the Progression  $1 + s + s^2 + s^3 \&c.$ ) is not so easy to determine; certainly it cannot be  $v = \infty \times 1$  or  $1 + 1 + 1 \&c.$  For then

$DEGBA = \infty \times \frac{s}{s-1}$ ; therefore if in one

Hyperbola we put  $s=2$ , (*scil.*  $y x^2=1$ ) we have  $DEGBA = \infty \times 2$ ; and putting in another  $1=3$  (*scil.*  $y x^3=1$ ) we have  $DGEBA = \infty \times \frac{1}{2}$ ; now this would make the Area of the Hyperbola  $y x^2 = 1$  *scil.*  $\infty \times 2$  greater than the Area of the Hyperbola  $y x^3 = 1$  *scil.*  $\infty \times \frac{1}{2}$ ; (because  $2 < \frac{1}{2}$ ) but it is easy to demonstrate, that in the Hyperbola's  $y x^s = 1$ . the greater we suppose the Exponent  $s$ , so shall the Area  $DEGBA$  (adjacent to  $BG$ ) be the greater, and consequently the last Term  $v$  of the Series  $1 + s + s^2 + s^3 \&c.$  cannot be  $\infty$  or  $1 + 1 + 1 + 1 + 1 \&c.$  for upon that Value of  $v$ , the Expression  $\frac{sv}{s-1}$  would make the Area  $DEGBA$  to decrease as  $s$  did increase.

To discover the Value of  $v$ , we have

$$DGEBA = \frac{sv}{s-1} = \frac{1}{1-s}; \text{ which gives } v$$

$$= \frac{s-1}{s-s} = \frac{s-1}{s} \times \frac{1}{1-s} = \frac{1}{-s}.$$

HAVING found  $v = \frac{1}{s}$ , we see that in two different Hyperbola's (*Ex. gr.*  $yx^2 = 1$ , and  $yx^3 = 1$ ;)  $v$  cannot have the same Value; for in the former  $v = \frac{1}{-2}$  which is less than  $v = \frac{1}{-3}$ , in the other.

*Secondly.* From the Value of  $v = \frac{s-1}{s} \times \frac{1}{1-s}$  we see that the last Term  $v$  is equal to the Sum of the Series (*viz.*  $1 + s + s^2 + s^3 \&c.$ ) multiplied upon  $\frac{s-1}{s}$  for  $\frac{1}{1-s}$  is  $= 1 + s + s^2 + s^3 \&c.$

*Corol.* I say the last Term  $v$  cannot be  $\infty \times n$ , take what Number you will for  $n$  (except in the Case\* of the common Hyperbola, where  $s=1$ .) For if it were possible, let  $v = \infty \times n$ ; then must  $\frac{s-1}{s-s^2} = \infty \times n$ , therefore  $s-1 = \infty \times ns - ns^2$ , which gives  $1 = \infty \times \frac{ns - ns^2}{s-1}$ , and consequently  $\frac{1}{\infty} = \frac{ns - ns^2}{s-1}$  but  $\frac{1}{\infty} = 0$ , therefore  $\frac{ns - ns^2}{s-1} = 0$  which give  $s=1$ . *Q. E. D.*

*Corol. 2.* It is evident that  $v$  is some Power of  $s$ , let the Exponent of that Power be  $n$ , *scil.*  $v = s^n$ , but  $\frac{s^v}{s-1} = \frac{1}{1-s}$ , therefore

$$s \times s^n$$

$\frac{s \times s^n}{s-1} = \frac{1}{1-s}$ , *id est*,  $\frac{s^{n+1}}{s-1} = \frac{1}{1-s}$ , therefore

$s^{n+1} = \frac{1}{s-1} \times \frac{1}{1-s}$ , which reduced to a

logarithmick Equation gives  $n+1 \times l: s-1:$

$s-1+l:\frac{1}{1-s}$ , which gives  $n = \frac{l:s-1+l:\frac{1}{1-s}}{l:s}$

-1. Q. E. I.

*An A N S W E R to Mr. Varignon's  
Reflections upon Spaces greater than  
infinite.*

**L**ET BC, AC making a right Angle  
at C be the Asymptotes of any Hyper-  
bola BGA; the Abscisse CL= $x$ , and Ordinate LG= $y$ ; and the general Equation to  
all Hyperbola's  $yx^e=1$ . Then because  $y =$

$\frac{1}{x^e} = x^{-e}$ , we have  $y dx = x^{-e} dx$ , therefore

$f: y dx = f: x^{-e} dx$ ; but by the *Calculus*

*Integralis*  $f: x^{-e} dx = \frac{x^{1-e}}{1-e}$ , therefore  $f: y$

$dx = \frac{x^{1-e}}{1-e}$ ; But  $f: y dx = BGLCB$  there-

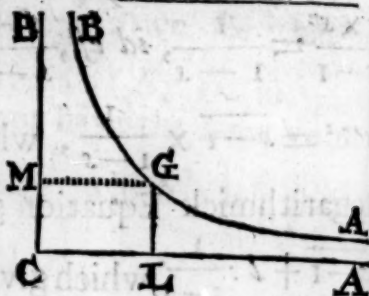
fore  $\frac{x^{1-e}}{1-e} = BGLCB$ ;

THIS



THIS Conclusion (though deduced naturally from Principles which Mr. *Varignon* acknowledges to be true) he denies, assuring us that it is only the negative Expression of the Area  $AGLA$  or Complement of  $BGLCB$ , when  $e < 1$ . But (without any Regard to the Ratio of  $e$  to 1) let us consider the algebraical Expressions of these two Spaces  $BGLCB$ , and  $AGLA$ ; we have already found  $BGLCB = \frac{x^{-e}}{1-e}$ ; and calling the Abscisse  $CM = y$ , and Ordinate  $MG = x$ , we shall from the Equation  $y x^e = 1$ , find  $ACMGA = \frac{e \times y \frac{x^{-1}}{e}}{e-1}$  but  $y^{\frac{e-1}{e}} = x^{1-e}$ ; therefore  $ACMGA = \frac{e \times x^{1-e}}{e-1}$ , from which if you subtract  $CMGL = yx = x^{1-e}$ , the Remainder will be (as Mr. *Varignon* says)  $AGLA = \frac{x^{1-e}}{e-1}$ , which is indeed the Negative of  $\frac{x^{1-e}}{1-e}$ .

But his Conclusion is false and groundless, *viz.* That  $\frac{x^{1-e}}{1-e}$  is not the Expression of the Space



Space  $BGLCB$ , but only (when  $e < 1$ ) the negative Expression of its Complement  $AGLA$ . For (according to the Principles of the *Calculus Integralis*)  $\frac{x^{1-e}}{1-e}$  is as certainly the

Expression of the Space  $BGLCB$ , as its Negative  $\frac{x^{1-e}}{e-1}$  is the Expression of its Complement

$AGLA$ . And this is suitable to Nature's Proceedings, *viz.* to do things in the simplest and most general Method, and consequently to give one Expression of the Area of a Figure and of its Complement, when it can be done. And may it not (with as good

Reason) be said that  $\frac{x^{1-e}}{e-1}$  is not the Expression

of the Space  $AGLA$ , but of its Complement  $BGLCB$ ; and not only it may be said, but Mr. *Varignon* must say so in all the innumerable Cases where  $e > 1$ ; and to say it is not when  $e < 1$ , and it is when  $e > 1$ ; what is this but to say backward and forward, according as it will answer his Conceit of denying that there are Spaces greater than Infinite? But to put this Controversy beyond all Dispute, let us resolve these two finite Expressions of the Spaces  $BGLCB$ , and  $AGLA$  into infinite Series's.

I.  $\frac{x^{1-e}}{1-e} = x^{1-e} \times 1 + e + e^2 + e^3 + e^4, \text{ \&c. when}$

you

you begin the Division with 1, as here it naturally should

$$\text{II. } \frac{x^{1-e}}{1-e} = x^{1-e} \times \frac{1}{e} - \frac{1}{e^2} - \frac{1}{e^3} - \frac{1}{e^4}, \&c.$$

When you begin the Division with  $-e$ .

$$\text{III. } \frac{x^{1-e}}{e-1} = x^{1-e} \times \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \frac{1}{e^4}, \&c.$$

When you begin the Division with  $e$ , as here it naturally should.

$$\text{IV. } \frac{x^{1-e}}{e-1} = x^{1-e} \times 1 - e - e^2 - e^3 - e^4, \&c.$$

When you begin the Division with  $-1$ .

Thus it appears, that each of these two Expressions give two Series, of which the first is the Area directly sought, and the other the negative Expression of its Complement; so

$$\left\{ \begin{array}{l} BGLCB = \frac{x^{1-e}}{1-e} = x^{1-e} \times 1 + e + e^2 + e^3 + e^4 + \&c. \\ -AGLA = \frac{x^{1-e}}{1-e} = x^{1-e} \times \frac{1}{e} - \frac{1}{e^2} - \frac{1}{e^3} - \frac{1}{e^4} - \&c. \\ AGLA = \frac{x^{1-e}}{e-1} = x^{1-e} \times \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \frac{1}{e^4} + \&c. \end{array} \right.$$

$- + \&c.$

$e^A$

$x^{1-e}$

$$-BCLCB = \frac{x^{1-e}}{e-1} = x^{1-e} \times -1 - e - e^2 - e^3 - e^4 - \&c.$$

These are plain and obvious Conclusions, deduced from the known and common Operations of Arithmetick, and utterly overthrow all that Mr. *Varignon* alledges against Spaces greater than Infinite.

For without any Limitation (except  $e < 1$ )

$x^{1-e}$

he says that — is not the Expression of

$1-e$

*BGLCB*, but the Negative of the Space *AGLA*. And if so, then (by Series I.) it will follow that  $x^{1-e} \times 1 + e + e^2 + e^3 + e^4, \&c.$  is the negative Expression of the Space *AGLA*, and this I say implies two manifest Absurdities; (1st) That a Sum of affirmative Quantities is negative, and (2dly,) That the Sum of an infinite geometrical Progression, whose Terms are continually increasing, is equal to a finite Quantity, *i. e.* That Affirmative is Negative, and Infinite is Finite.

That which seems to shock Mr. *Varignon* is, that there should be Spaces greater than Infinite; for he tells us expressly, that he looks upon this as a Contradiction. But this Difficulty will soon vanish, if he considers

ders what is the simple Idea of Infinite ; and because he confesses that in the *Apollonian* Hyperbola  $BGA$ , the Space  $BGLCB$  is Infinite ; therefore let us take the arithmetick Value of  $BGLCB$  for infinite, but in that Hyperbola  $e=1$ , and therefore (if we put likewise  $x=1$ ) we shall by Series 1 have  $BGLCB = 1 + 1 + 1 + 1 + 1$  &c. that is  $BGLCB = \infty \times 1$  or an infinite Number of Units. So that when we say  $2 + 2 + 2 + 2$  &c. or  $\infty \times 2$  is more than infinite, no more is meant, but that it is greater then simple Infinite or  $\infty \times 1$  ; *i. e.* (taking  $\infty \times 1$  for the Unit, by which we measure Quantities that are infinite)  $\infty \times 2 < \infty \times 1$ , or  $2 < 1$ .

This will be yet plainer in the Case of Finites. It is certain no Quantity can be less than Finite (as none can be greater than Infinite) which we shall denote by  $\odot \times 1$  or 1 ; when I say then that  $\odot \times \frac{1}{2}$  is less then  $\odot \times 1$ , I mean no more but that  $\frac{1}{2} < 1$  ; and because I have defined the Idea of finite by  $\odot \times 1$  or 1, I may very properly say that  $\odot \times \frac{1}{2}$  or  $\frac{1}{2}$  is less than a finite Quantity.

September 23d, 1713.



F I N I S.



## Advertisement.

THE Errors of the Press, in the *first Part*, not being such as can stop the *intelligent Reader*, I beg he will mend the following ones, of the first and second Chapters of the *second Part*, which were the most material I could observe. *Page 8. Def. 9. relative Nothing* is said here to be generated by a perpetual *Subtraction*, tho' the Signs be alternately + and —. For these Reasons, because *relative Infinite*, was said to be generated by a perpetual *Addition*, and because that after the first Term, every two succeeding ones in *relative Nothing* *is equivalent to 0* I thus  $1 - 1 + 1 - 1 + 1 - 1, \&c.$  is  $1 - 1 + 1 - 1 + 1 - 1 + 1, \&c. = 1 - 01 - 01 - 01 - 01 \&c.$  and so in other Cases, *p. 22. line 10.* for effect read affect: *p. 27. l. 10.* for  $\infty^{n-1} : n :: 1 : 0. r. \infty^{n-1} \infty^n :: 1 : 0.$  *p. 30. l. 7.* for Indefinite, *r. Infinite.* *p. 32.* The Errors of Calculation here are corrected. *p. 163. p. 41. l. 13.* instead of Distance, *r. Time* employed in the Motion of Light. *p. 42. l. 12.* instead of waies *r. Times.* *p. 84. l. 23.* instead of Relations, *r. Revelations.* *p. 101. l. 17.* instead of are essentially, *r. are not essentially.* *p. 108. l. 21.* instead of Limit, *r. Light.* The Errors of the *third Chapter*, will not stop those who would otherwise understand it.

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